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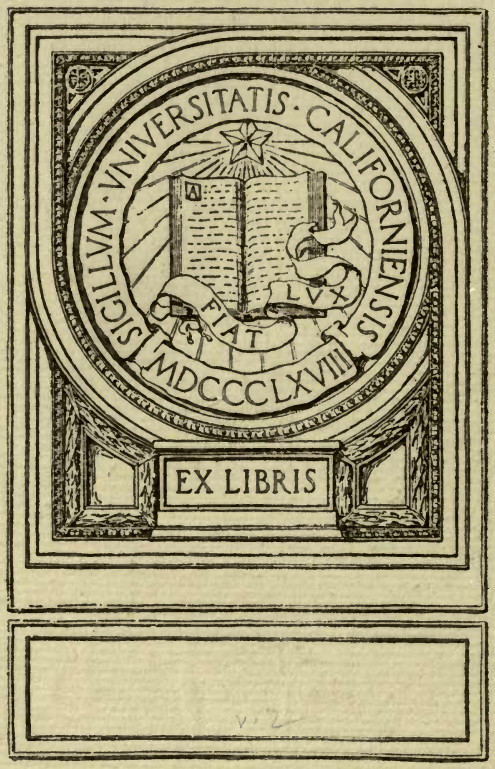
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# A TEXT-BOOK OF PHYSICS

# A TEXT-BOOK OF PHYSICS

I.

SOUND

II.

LIGHT

BY

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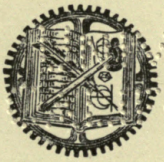

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# PREFACE

This book was originally issued in one volume, but at the request of many teachers it is now divided into three parts. No alterations have been made except a slight re-arrangement of appendices.

The preface to the whole work appears in Volume I., and explains the origin and scope of the book. The authors wish, however, to take this opportunity of again thanking those whose help has in many ways lightened the task of preparing the subject matter and seeing it through the press.

R. T. L.
H. E. H.

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# CROSS-REFERENCES

THE following system of cross-references has been employed throughout the book.

References are to §§. Sections are denoted by the following initials:—

| | |
|---|---|
| D = Dynamics. | M = Magnetism. |
| H = Heat. | ES = Statical Electricity. |
| S = Sound. | EC = Current Electricity. |
| L = Light. | |

Thus, "§ 21, D" means § 21 in the section DYNAMICS.

Where no initial is given the reference is to a paragraph in the same section of the book in which the reference occurs.

# A TEXT-BOOK OF PHYSICS

## SOUND

### CHAPTER I

#### WAVE MOTION

§ 1. *Simple Oscillation.*—If a leaden bullet be hung up by a thin string and then displaced from its position of equilibrium it will swing from side to side, and moreover if its swings be timed it will be found that, provided they are not too large, they will each take place in the same time. Such vibrations are called isochronous. When the bullet is displaced the earth tends to pull it back to its position of equilibrium, and on its being released the attraction of the earth makes it move faster and faster, until it reaches its lowest point, then the momentum it has gained carries it onward and upward against the force due to the earth, which now retards it, until it reaches a point nearly as high as before, and then the oscillation is repeated. The time of a complete oscillation is reckoned from any point of the path of the pendulum-bob until the bob is again at the same point moving in the same direction. Thus from the time the bob is let go until it reaches its highest point on the other side is half an oscillation. Observation shows that, in the case of the simple pendulum described, the time of a complete oscillation at any particular place depends only on the length from the support to the middle of the bob. The number of oscillations per second is sometimes called the frequency, and the time of an oscillation is called its period. The greatest distance the bob moves from the equilibrium position is its amplitude.

§ 2. *Wave Model.* — The simple model about to be described illustrates the connection between a simple oscillation, like that of a pendulum, and wave motion.

A number of pendulums having strings of equal lengths are suspended from a horizontal bar as in Fig. 1. They can all be displaced equal amounts by means of a vertical board of the shape

shown. When the board is dropped the pendulums start to swing one after the other, beginning from the left, because as the board drops each pendulum is released before its right-hand neighbour. The appearance presented is that of a wave travelling along the row of pendulums, and from a study of what happens in the case of this model we can learn the principal characteristics of wave motion.

The first point to be observed is that each succeeding pendulum is moving in exactly the same way as the one before it, but is

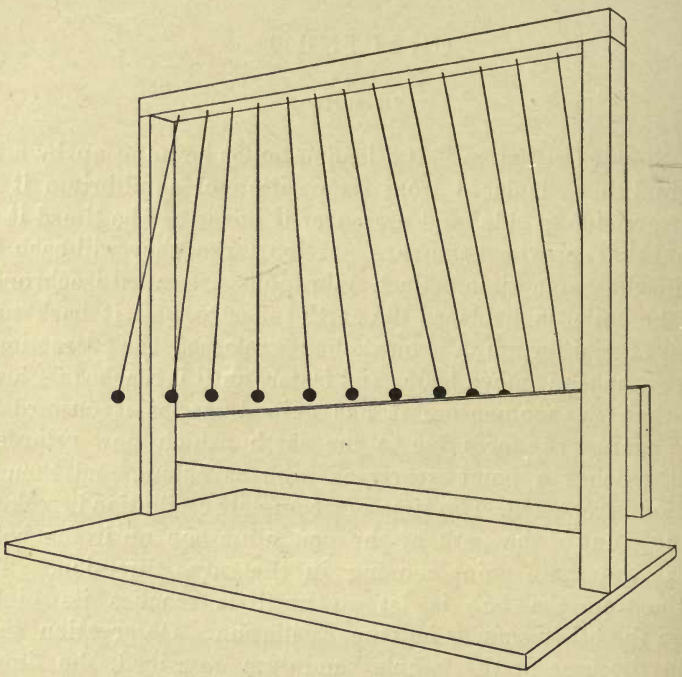


Fig. 1.

always a little behindhand in its movements, or, we may say, is a little different in phase.

Plans of the positions of the pendulum bobs at succeeding times, viewed from above, would look something like Fig. 2.

The original displaced position of all the bobs is first shown. A shows the position when the first bob has swung back as far as its equilibrium position. B shows that when the first bob has swung out to the other side the seventh bob, say, is just about to move. C shows the state of affairs a quarter of a period later, and

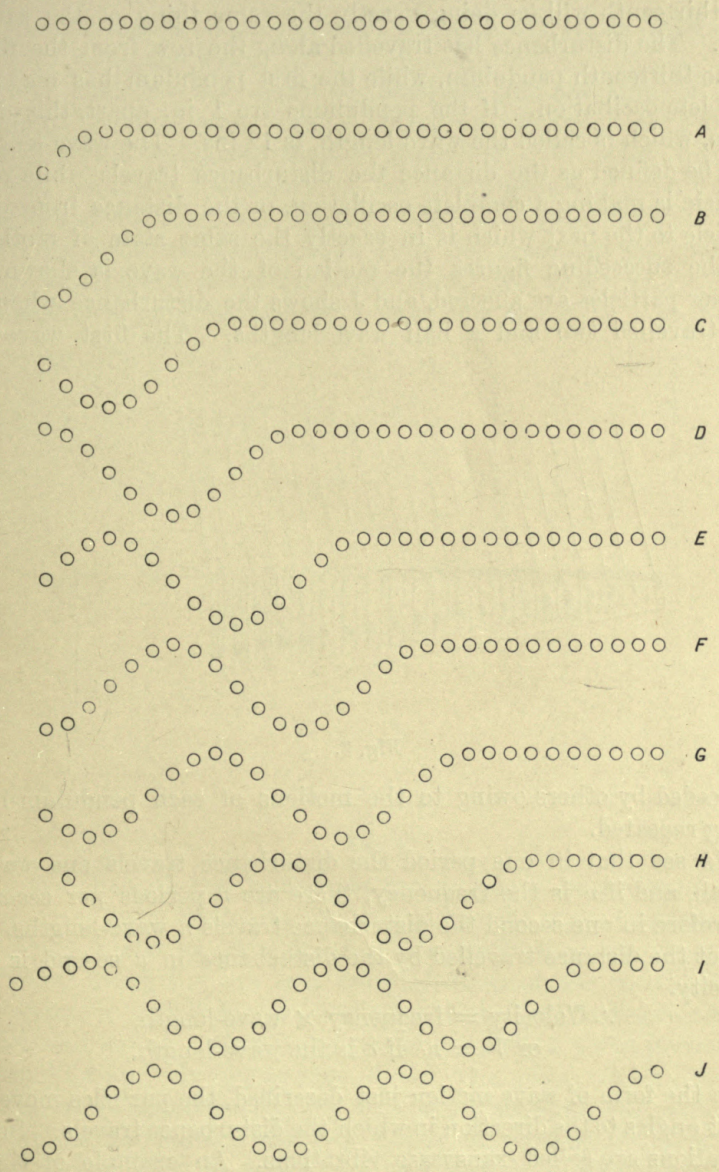



Fig. 2.

in D the thirteenth bob is just about to start its first oscillation, while the first bob is just about to start its second; and so the first

and thirteenth will be doing exactly the same thing at the same time. The disturbance has travelled along the row from the first to the thirteenth pendulum, while the first pendulum has made a complete oscillation. If the pendulums are 1 in. apart, this distance, which is called the wave-length, is 12 ins. The wave-length may be defined as the distance the disturbance travels while one particle is making a complete oscillation, or the distance from one particle to the next which is in exactly the same state of motion. In the succeeding figures the motion of the wave is shown as further particles are affected, and J shows the disturbance when it has travelled two and a half wave-lengths. The first wave is

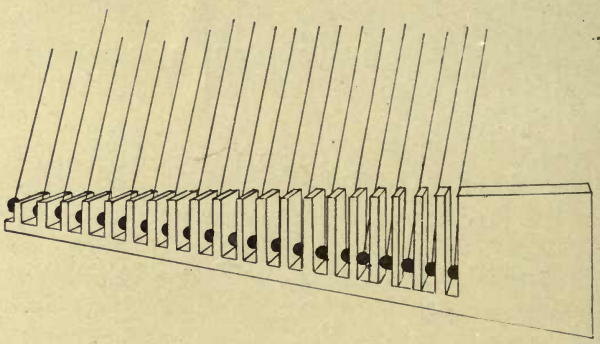


Fig. 3.

succeeded by others owing to the motions of each pendulum-bob being repeated.

We see that in one period the disturbance travels one wave-length, and if $n$ is the frequency, there are $n$ periods per second. Therefore in one second the disturbance travels $n$ wave-lengths.

But the distance travelled by the disturbance in a second is its velocity.

$$\therefore \text{Velocity} = \text{frequency} \times \text{wave-length},$$
$$\text{or } V = n\lambda \text{ if } \lambda \text{ is the wave-length.}$$

In the form of wave motion just described, the particles move at right angles to the direction in which the disturbance travels. Such vibrations are called transverse vibrations. An example of these is seen when a gust of wind passes over a wheat field. Each individual stalk is bent down and then straightens itself, and as each succeeding one does the same thing at a time slightly later, a

depression travels across the field, although the stalks themselves only move up and down.*

We could use the model to illustrate another kind of wave motion. In this each particle makes an oscillation in the line along which the disturbance is travelling. To start this we should require a framework of the shape shown in Fig. 3. By displacing all the bobs equally and then dropping the wood each one would start to oscillate after the preceding one, and so a wave would travel along the row of bobs. Let us try by the aid of a diagram to see what kind of a wave this would be. In Fig. 4

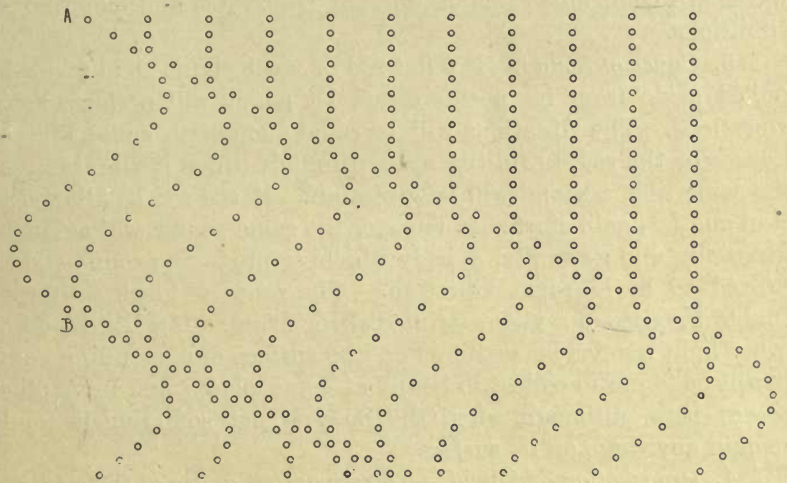



Fig. 4.

the first line of dots represents the displaced bobs viewed from above, and each successive line represents the state one twentieth of a period later let us suppose. The disturbance which appears to travel along the row is a crowded state of the bobs, followed by a state in which they are further than the normal distance apart. Each bob, however, simply executes an oscillation about its position of equilibrium. It will be noticed that at a place where the bobs are crowded together they are all moving forward, and at a place where they are scantily distributed they are moving in the opposite direction to that in which the disturbance is travelling. The vibrations in this case are said to be longitudinal.

In the state represented by B, the first particle has made a

* There is also a slight motion to and fro.

complete vibration and the eleventh particle is just about to go through the same motions with the first, so the wave has travelled over ten times the original distance between the particles. This distance is the wave-length as previously defined. When long waves are formed on shallow water the motion is very nearly of this kind, as the particles describe very elongated ellipses with their major axes in the direction of propagation. In waves on deep water the particles move in circles whose planes are vertical and parallel to the direction in which the wave is moving. Thus the particles do not travel far from their average position, and the waves are compounded, as it were, of transverse and longitudinal vibrations.

§ 3. *Cause of Sound.*—If a flat rod of steel clamped in a vice be pulled aside from its position and let go, it will perform rapid vibrations. The vibrations will be rapid enough to cause blurred images of the rod to fill the space through which it vibrates. At the same time a sound will be given out. If the rod be allowed to stop and be again started to vibrate, the same sound will again be given out, and generally, a body which is producing sound can be proved to be in rapid vibration. The sense of touch will prove this in the case of a violin string, bell, or drum. If a little sand or other light powder be scattered on the surface of a sounding body, it will be found to collect in heaps at some places and leave other places bare, although when the body is not sounding it would remain anywhere on its surface.

§ 4. *Propagation of Sound.*—To return to the case of the clamped rod; suppose the rod has been displaced and just let go. As it moves back it leaves a partial vacuum behind it into which some of the air is forced. Thus there exists a portion of the air rarer than that surrounding it. Presently the rod reaches its extreme position on the other side and commences to move back, compressing the air in front of it, and the process is repeated. Now let us see what happens to the air particles.

As the rod starts to move from right to left, the layer nearest it expands to fill the vacuum created and so its pressure is diminished. Thus we have a layer of air in which the pressure is less than that in the layer to the right of it, hence the layer to the right expands in turn, and so we get every layer successively expanding, or in other words a rarefied condition travels through the air. It is obvious that, in a rarefaction, on the average the particles are

moving in the opposite direction to that in which the disturbance is propagated.

When the rod commences to move back it compresses the layer next to it, and this tending to regain its normal state expands and compresses the layer to its right, and so all layers are compressed in turn, and a compression travels through the gas. As the motion of the rod is repeated we get a series of rarefactions and compressions travelling after each other through the gas. In a compression the particles of air are moving forward, so the motion we are considering is very like that of the pendulum-bobs in the second

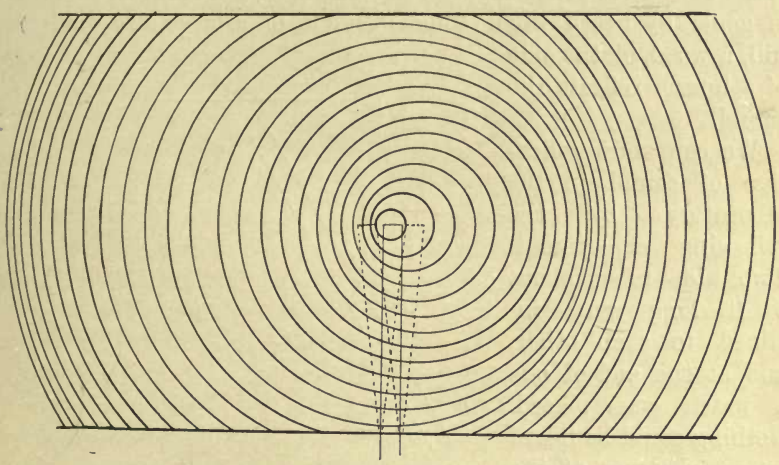


Fig. 5.

case. It is these waves of compression and rarefaction travelling through the air which produce the phenomena of sound. From what we have said it will be seen that each particle moves backwards and forwards, but does not travel far. What travels through the air is a condition of compression and rarefaction. The state of the air at any instant may be represented by Fig. 5, where the lines close together represent a compression.

It is obvious that, if on one side of the vibrating rod there is a compression, on the other side there is a rarefaction, and these travel out into the air around.

§ 5. *Reflection of Sound.*—Suppose a single compression of the air sent out by the vibrating bar meets an obstacle with a plane surface—a wall, for example. Then in the layer of air next the wall

we have a greater pressure than in layers behind it, and the air in this layer expands, compressing that in the layer behind, and, in exactly the same way as we have explained before, the compression travels back through the air. So that the single compression on meeting the wall travels back, or, in other words, is reflected. When a rarefaction reaches the wall the air in the layer behind expands to equalise the pressure, and thus this layer is rarefied, and so on for layers still further back. Thus a rarefaction is reflected. Hence, if waves of sound, which consist of compressions and rarefactions following each other, fall on a wall they will be reflected. This reflection of sound gives rise to echoes. An echo is generally heard when some obstacle of a large size, such as a hill, a wood, or the side of a house, reflects the sound.

We can show reflection of sound in the laboratory by the following experiment. Two wide tubes about 6 ft. long were inclined to each other, and at the end of one a watch was placed. Nothing could be heard by an observer at the end of the other tube, but when a piece of board was placed opposite the open ends of the tubes, so as to make approximately equal angles with the tubes, the ticking was distinctly heard by an observer with his ear placed against the receiving tube. For a plan of the apparatus see Fig. 6.

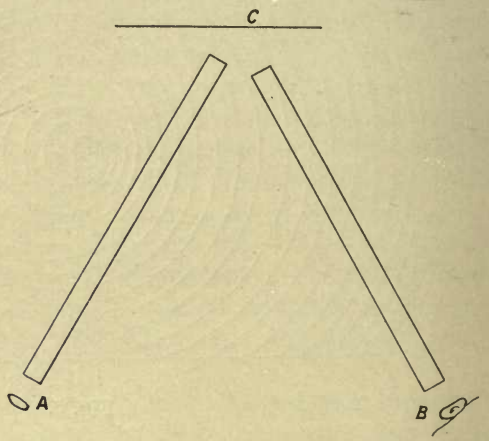


Fig. 6.

If the board is not equally inclined to the two tubes the ticking will not be audible, thus proving that sound is reflected according to the same law as light and heat. The tubes make the effect more intense, because the waves of sound travelling through the air in them are prevented from spreading out in all directions as they would in the open air. When the waves meet the sides of the tube they are reflected, and the sound is concentrated, so to

speak. The same effect takes place with an ear trumpet. As a wave of sound travels along the trumpet the motion of the air particles in one layer is communicated to those in the next, but as the tube narrows there are fewer and fewer particles in each succeeding layer, and so the motion of the particles gets greater, since the motion of one set of particles is passed on to a smaller number. It is the extent of motion of the particles which determines the loudness or intensity of the sound. Thus, the harder the membrane of a drum is struck, the greater will be the extent of its motions, and so the louder will be the sound produced, for the oscillations set up in the air will be more violent.

A good illustration of a state of compression travelling can be obtained by taking some thin brass wire and coiling some hundreds of turns on a glass tube about an inch and a quarter in diameter. When the turns are slipped off the tube we have a spiral spring. One end of the spring is fixed in a vice or hung from a ceiling and the spring allowed to hang down. The loose end is taken in the hand, and thirty or forty of the end coils are pressed close together while the rest of the spring is stretched so as to bring the coils in it about a quarter or half an inch apart. On releasing the compressed coils and retaining the end of the spiral, the compressed state travels rapidly upwards, reaches the top where it is reflected, travels downwards, and if it has not died away may be again reflected at the bottom. It will be noticed that the compression does not spread itself equally along the wire when released, but travels as a whole. Exactly the same thing takes place with waves of compression in air. A rarefaction can be made to travel along the spring in exactly the same way, but it is more difficult to observe.

§ 6. *Sound is not Produced in a Vacuum.*—Matter is necessary for the propagation of sound. This can be proved by a well-known experiment. A bell and a clockwork arrangement for striking it are suspended by threads inside a bell jar which stands on the plate of an air-pump. The bell can be distinctly heard outside. The pump is then used until the pressure of the air has been brought very low, when the sound is practically inaudible. A very small quantity of air introduced, however, is sufficient to render the sound again audible. The slight sound which is heard is due to the transmission of vibrations by the threads which suspend the bell.

*Examples.*

1. A heavy bob, hanging in equilibrium at the end of a light thread, is struck sideways: describe and explain in general terms the resulting motion, illustrating by the terms *period*, *frequency*, *amplitude*, *phase*, *isochronism*. (Oxf. Prelim. and C. (3) 04.)

2. What is meant by a simple harmonic vibration? Sketch the curve in which the ordinate represents the displacement in such a vibration at a time represented by the abscissa. (Army 04.)

3. Explain what is meant by the wave-length of a musical note. Give a diagram showing the changes in the compression and rarefaction of the air which occur in a wave length. (Army 00.)

4. Describe an experiment to show that in ordinary cases the air is the body chiefly concerned in transmitting sounds. (Oxf. Loc. Jun. 07.)

5. Give reasons for believing that sound is propagated through air as waves of compression and rarefaction. (Army 03.)

6. Explain with diagrams the method of propagation of sound in air, and define the terms *wave-length* and *vibration-frequency*.

If you knew the velocity of sound and the frequency of a particular note, how would you calculate its wave-length? (Oxf. Prelim. and C. (3) 07.)

7. What do you understand by wave-motion? Does the velocity of sound waves in air depend on the wave-length? Give reasons for your answer. (Camb. Loc. Jun. 07.)

8. Two short lengths of tube, $A$ and $B$, are taken; over one end of each a piece of parchment is stretched, and the centres of the two pieces of parchment are connected by a long thin stretched wire, the whole forming a toy telephone. Give a general explanation of the manner in which a sound produced near $A$ causes $B$ to give out a sound. (Camb. Loc. Sen. 06.)

9. Describe an experiment to illustrate the reflexion of sound. How could you use this experiment to determine the velocity of sound? (Camb. Loc. Jun. 05.)

10. Explain the formation of an echo and of a sound shadow. Would you expect one of these to occur without the other? Give reasons for your answer. (Camb. Loc. Sen. 07.)

## CHAPTER II

### VELOCITY OF SOUND

§ 7.—In the model which we have employed to illustrate wave motion we can alter the velocity of the wave at will. The more slowly the board which frees the pendulums is let down, the longer is the interval between the motion of one pendulum and the next, and hence, during an oscillation of the first pendulum, fewer pendulums will be set in motion, and so the wave will not travel so far. In the case of a wave travelling through a fluid the particles in each layer cause those in the next to move by reason of their own motion. One layer is compressed or rarefied and passes this condition on to the next. The rate at which this condition travels is the velocity of sound. When one layer is compressed, by regaining its ordinary pressure it compresses the next, and the more quickly the first recovers, the more rapidly will it pass on the compression. So the velocity of sound depends on the rapidity with which a compressed layer can recover and compress the next layer.

The rapidity of recovery from compression depends on two things, the density of the layer of gas which is to be compressed, and the elasticity of the layer which is at present compressed. The greater the density of a gas, the harder it is to compress; for instance, if we close the end of a bicycle pump and compress the air inside, at first the piston goes in easily, but as the volume of gas gets less and its density correspondingly greater the work becomes harder. We notice, too, that compressing the gas causes the development of heat, for in pumping up a tire, where at each stroke we compress the air, the pump gets appreciably hot, and this heat is not due to friction, for it is not developed if we leave the end of the pump open and just move the piston up and down.

So far, then, we have established that the velocity of sound depends partly on the rapidity with which a layer of gas can be compressed, and that as its density increases a gas becomes more difficult to compress. Hence, increase of density will tend to diminish the velocity of sound.

Now let us see how the elasticity of a substance influences the quickness of its recovery from compression. Suppose we have a volume $V$ of a substance which is subjected to a uniform pressure $P$. If the pressure is increased the volume will be diminished, and the increase in pressure being small ($p$, say), the decrease of volume $v$ will be small also.

The ratio of the decrease of volume to the original volume is $\frac{v}{V}$, and the elasticity is defined as the ratio of the small increase of pressure to this small fractional decrease of volume, *i.e.*,

$$\text{Elasticity} = \frac{p}{\frac{v}{V}} = V\frac{p}{v}.$$

If the change of volume is small, as we have stated in our definition, the body will return to its original volume on restoring the pressure to its original value, and the smaller the change of volume produced, the more rapid will be the return.

A very elastic body will be one in which a large increase of pressure only produces a slight diminution in volume, and such a body will recover its original volume rapidly after a compression. A glass marble, for instance, is very elastic, for it requires a large compressing force to change its volume, while a tennis ball is not so elastic.

Hence, the greater the elasticity of a fluid the more rapidly will a layer of it recover from compression, and hence the greater the velocity of sound in that fluid.

The velocity of sound, then, increases as the elasticity increases, and decreases when the density increases. By the aid of mathematics it can be deduced that the velocity of sound or of any wave of compression and rarefaction in a gas is

$$U = \sqrt{\frac{E}{\rho}},$$

where $E$ is the coefficient of elasticity and $\rho$ is the density.

§ 8. *Elasticity of a Gas under Different Conditions.*—Returning to the illustration of the bicycle pump, we can either compress the air in it so rapidly that there is not time for the heat to escape, or so slowly that heat escapes by conduction and the air does not rise in temperature. The first of these is called Adiabatic compression,

and the second Isothermal compression. It is obvious that the elasticities in these cases will be different. For suppose we increase the pressure and compress the gas isothermally. At the end of the operation the gas is at the same temperature as at the commencement. Had we applied the same increase of pressure and compressed it adiabatically—that is, allowing no heat to escape—at the finish the gas would have been under the same pressure, but at a higher temperature than before. Thus we have the same mass of gas under the same pressure, but at a higher temperature in the second case than in the first. Therefore the volume in the second case is greater than in the first. That is, the application of increased pressure isothermally produces a greater diminution of volume than the same increase of pressure produces when no heat escapes. Therefore, from the definition of elasticity,

$$E = V\frac{p}{v},$$

since $V$ the original volume and $p$ the increase of pressure are the same, and $v$ is greater for isothermal compression than for adiabatic, the adiabatic elasticity is greater than the isothermal elasticity.

The question arises, which elasticity is to be taken in the formula $U = \sqrt{\frac{E}{\rho}}$? When sound travels through a gas the compressions and rarefactions take place so rapidly that there is no time for the heat to escape from the portion of gas in which it is produced, and so we must take the adiabatic elasticity. For air the adiabatic elasticity is found to be about 1·41 times the isothermal elasticity. It is not difficult to calculate the elasticity at constant temperature; for we know that, when the temperature remains constant, by Boyle's law (§ 84, D.) the volume varies inversely as the pressure, or $PV = \kappa$ where $\kappa$ is a constant.

Let $PV$ be the volume and pressure of a quantity of gas, and let the pressure be increased by $p$, while the volume decreases by $v$, both $p$ and $v$ being small compared with $P$ and $V$.

$$\text{Then } PV = \kappa \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the new pressure is $P + p$, and the new volume $V - v$.

$$\text{Therefore } (P + p)(V - v) = \kappa,$$

$$\text{that is, } PV + pV - Pv - pv = \kappa \quad . \quad . \quad (2)$$

but from (1) substituting for $PV$

$$pV - Pv - pv = 0,$$

and $p$ and $v$ are both small. Therefore we can neglect the product $pv$ in comparison with the other terms and we get

$$pV - Pv = 0,$$
$$\text{that is } pV = Pv,$$
$$\text{or } V\frac{p}{v} = P.$$

But $V\frac{p}{v}$ is the elasticity.

Therefore the isothermal elasticity of a gas is equal to its pressure.

The velocity of sound is $\sqrt{\frac{E}{\rho}}$ where $E$ is the adiabatic elasticity, which is equal for air to 1·41 times the isothermal elasticity or to 1·41 times the pressure.

$$\text{Hence } U = \sqrt{1{\cdot}41 \times \frac{P}{\rho}}.$$

The formula $\sqrt{\frac{E}{\rho}}$ for the velocity of sound was first found by Newton, but the elasticity was taken as the isothermal elasticity, giving too small a result for the velocity of sound. Laplace first discovered that the adiabatic elasticity should be used, and then the calculated result was found to agree with the result of experiment. The velocity in air at 0° C. is about 330 metres per second or approximately a kilometre in 3 secs., or 1,086 ft. per second.

§. 9. *Effect of Change of Pressure or Temperature.*—We can write the formula for the velocity of sound in a different form. Let $V$ be the volume occupied by a gram of air at pressure $P$. Then since density is $\frac{\text{mass}}{\text{volume}}$, $\rho = \frac{1}{V}$,

$$\therefore U = \sqrt{1{\cdot}41\,\frac{P}{\rho}} = \sqrt{1{\cdot}41\,PV},$$

but $PV$ is constant provided the temperature does not change.

If the temperature changes and the pressure remains at $P$, we know that if $V_o$ is the volume of 1 gram of air at pressure $P$ and 0° C., and $V_t$ its volume at the same pressure and at temperature $t°$,

$$V_t = V_o\left(1 + \frac{t}{273}\right) \text{ (see § 41, H.).}$$

The velocity of sound in air at pressure $P$ and 0° C. is $\sqrt{1{\cdot}41\ \frac{P}{\rho_o}}$ or $\sqrt{1{\cdot}41\ PV_o}$, and in air at $t°$ C. it is $\sqrt{1{\cdot}41\ PV_t}$.

This is $\sqrt{1{\cdot}41\ PV_o\left(1+\frac{t}{273}\right)}$.

The velocity at $t°$ C. is therefore the velocity at 0° multiplied by $\sqrt{1+\frac{t}{273}}$.

The velocity at 0° C. is about 330 metres per second, and the velocity at 1° C. is $330\sqrt{1+\frac{1}{273}}$ metres per second, that is, $330\left(1+\frac{1}{546}\right)$ approximately. So that the velocity increases by about 0·6 metre per second for 1° rise of temperature.

§ 10. *Experimental Determination of Velocity of Sound in Air.*—This can be determined approximately by measuring the interval between the production of a sound such as the report of a cannon, and its perception by an observer at a distance. Since light travels at the rate of 186,000 miles per second (*see* §§ 2, L., 90, L.), the observer sees the flash almost at the time it is produced, but does not hear the sound for some time after. The interval can be measured by means of a stop-watch, and the distance divided by this time gives the velocity of sound. There are, however, sources of error here, the principal one being that it takes an appreciable time for a sound to be heard and the observer to record it by stopping a watch. To lessen this the observer can be replaced by a recording mechanism. A piece of blackened paper is made to move at a uniform speed by means of a revolving drum, and on this a pendulum makes a trace every second, or half second. Across the muzzle of the cannon is stretched a wire which forms part of an electric circuit in which is an electro-magnet and an iron pole-piece suspended by a stretched spring. On the pole-piece is a tracing point which traces a straight line on the paper. When the cannon is fired the wire across its mouth is broken, the current is stopped, and the pole-piece on being released causes the tracer attached to make an irregularity in the line. The receiver of the sound is a stretched rubber membrane with a thin metallic disc attached. The sound causes the membrane to vibrate, which causes the disc to complete or break another electric circuit with a magnet and tracing point, by coming into contact with metal terminals. The

moving paper is thus marked at two places, one of which represents the time of production of the sound, the other its time of arrival at a distant station. The pendulum makes marks on the paper representing seconds, and by comparing these with the marks made by the sound, the time at which the sound starts and the time at which it arrives can be found.

Fig. 7 gives a rough plan of the arrangements. A is the cannon, B the receiving membrane, C D the electro-magnets, E the moving paper, F F the two tracing points, and G the path of the pendulum which leaves a trace at its lowest point. From the distance between the two traces made by the production and arrival of the sound the time taken can be estimated to a small fraction of a second.

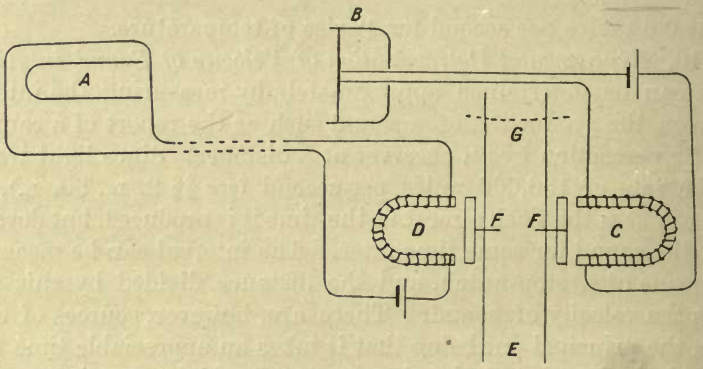



Fig. 7.

We have seen, however, that change of temperature affects the velocity of sound. Thermometers were therefore placed at intervals along the path of the sound, and the mean of the temperatures recorded by these was taken as the average temperature. The presence of water vapour in the air also affects the result, and so does a wind. It is well known that sound travels better with the wind than against it. The explanation of this will be given later. To eliminate errors due to wind, observations were taken with the sound travelling first in one direction and then in the opposite direction.

§ 11. *Velocity of Sound in Water.*—This has been determined by anchoring two boats some distance apart on a lake. A bell was suspended from one under water, and a lever was arranged so as to strike the bell, and at the same time ignite some gunpowder by means of a lighted wick at the other end. The sound was received

at the other boat by means of a large trumpet whose end, covered by a membrane, was under water, the other end being placed at the ear of an observer. The time between seeing the flash and hearing the sound gave the time taken to travel the distance between the boats, and hence the velocity.

Practically no heat is developed in compressing water,* so there are not two elasticities, and it is interesting to note that the velocity calculated from the formula

$$U = \sqrt{\frac{E}{\rho}}$$

agrees with that obtained by experiment. At 8° C. the velocity

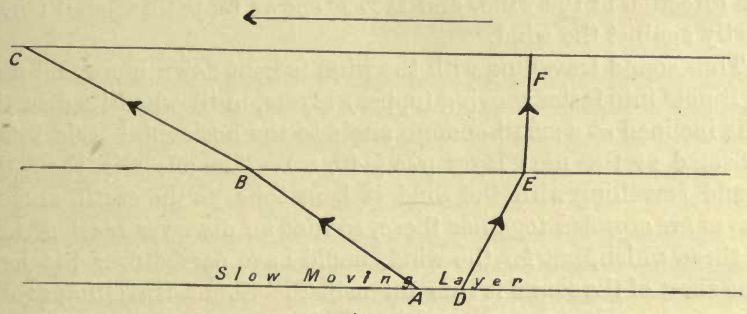



Fig. 8.

was found to be 1,435 metres per second; about four and a half times as great as the velocity in air.

Sound is transmitted with great rapidity in most solids. This is shown by the experiment of listening with the ear against a telegraph post while a further one is struck sharply. Two sounds are heard—the first, from the post, has travelled through the earth; and the second, which occurs appreciably later, through the air. The greater distance at which footsteps can be heard when the ear is placed against the ground is an example of the transmission of sound through a solid.

§ 12. *Effect of Wind.*—Sound travelling in the direction of a wind is heard better than sound travelling against the wind. The reason of this is not that the wind blows the sound along. Let us suppose a sound to be made in still air. It travels out in all directions at the same speed, so that if we wished to indicate the position

* Water is only very slightly compressible, and for most purposes may be considered to be incompressible.

of a compression at any time we should draw a sphere round the origin of the sound, and at all places on this sphere there would exist a state of compression at this time. Immediately afterwards there would be a rarefaction at the same place, and so on. Such a sphere would be called a wave surface, and the waves of sound would be spherical waves.

If a wind is blowing, the layers of air nearest the ground are moving slowest, and it can be proved that sound is altered in direction when it passes from a slow moving layer to one moving faster. The way in which this takes place is shown in Fig. 8, where A B C shows the path of a sound when the sound travels partly in the direction of the wind, and D E F shows its path when it travels partly against the wind.

Thus sound travelling with the wind is bent down more and more as it gets into faster moving (upper) layers, until when its direction gets inclined at a small enough angle to the horizontal it is totally reflected at the next layer (*see* § 40, L.). The effect is, then, that sound travelling with the wind is bent towards the earth, and the waves are crowded together there, so that an observer receives some of those which, but for the wind, would have passed over his head. The effect of the sound is thus intensified. Sound travelling against the wind tends to rise, as Fig. 8 shows, and thus what would ordinarily reach an observer passes up into the air above him.

*Examples.*

1. Explain how the velocity of sound may be determined (1) by experiments made in the open air; (2) by experiments made in a laboratory. (Army 01.)

2. State the law connecting the velocity of sound through a gas with its temperature and pressure. If the velocity of sound through hydrogen at 0° C. is 4,200 ft. per second, what will be the velocity of sound (at the same temperature) through a mixture of two parts by volume of hydrogen to one of oxygen? Will the velocity be the same as through water vapour?

The density of oxygen is 16 times that of hydrogen. (Army 01.)

3. Describe a laboratory experiment by means of which you could determine the velocity of sound in air, given a tuning fork or other means of producing a note of known pitch.

If the velocity of sound in nitrogen is 1,110 ft. per second, what would be the velocity in hydrogen at the same temperature? (Density of nitrogen : density of hydrogen = 14 : 1.) (Army 03.)

4. What is an echo? Why is a succession of echoes sometimes observed?

A man fires a gun on the seashore in front of a line of cliffs and an observer,

equidistant from the cliffs and 300 ft. away from the firer, notices that the echo takes twice as long to reach him as does the direct report. Find, by calculation or graphically, the distance of the men from the cliffs. (Army 04.)

5. What is the nature of the motion of air when sound is travelling through it?

Describe any experiment for determining the velocity of sound, (*a*) in air, (*b*) in water. (Oxf. Loc. Jun. 08.)

6. Discuss the effects of pressure, temperature, and wind on the propagation of sound waves in air. (Army 05.)

7. Two sounds of the same intensity sometimes on different days travel to very different distances; how do you explain this? (Army 00.)

8. Being given that the velocity of sound in any medium is inversely proportional to the square root of the density, show how to determine the effect of rise of temperature on the velocity of sound in the open air.

The velocity of sound at 0° C. is 1,100 ft. per second. At what temperature will the velocity be 1,210 ft. per second? (Matric. 08.)

9. How has the speed of sound in water been experimentally determined?

If the speed of sound is 1,120 ft. per second in air and 4,704 ft. per second in water, find the vibration frequency and the wave-length in water of the sound whose wave-length in air is 5 ft. (Oxf. Prelim. and C. (3) 07.)

10. What is the physical explanation of the differences observed between musical sounds?

A man walks away from a high wall striking a board with a hammer once every second. How far must he go from the wall before he hears the echo of one stroke simultaneously with the next stroke? (Oxf. Prelim. and C. (3) 08.)

## CHAPTER III

### CHARACTERISTICS OF MUSICAL NOTES

§ 13.—We are familiar with two kinds of sound, which we call music and noise. The special property of a musical sound is that it does not change its character. If we sound a note on the piano the sound remains the same, however long it continues, although it may diminish in loudness. On the other hand, what we call noise is either a sound which changes its character as it continues, or else is in the nature of a sudden crack or explosion. Both of these kinds of noise can be heard in a thunderstorm—in the soughing of the wind and the crack of the thunder. The rattle of a badly-fitting window shaken by the wind is another example of the first, while the noise of a stick breaking is of the second kind.

§ 14. *Experiments on the Production of Musical Sound.*—For this we shall require a whirling table and a large nut with a milled edge. The whirling table consists of a large wheel with a grooved edge which can be turned by hand, joined up by a belt to a much smaller wheel, to the axle of which can be fixed anything which is to be rotated.

On the table used for the experiment about to be described the large wheel had a diameter six times as large as the small wheel, consequently the small wheel revolved six times while the large one revolved once. The instrument is shown in Fig. 9. The axis of the small wheel has a shoulder formed on it, and the thinner part is threaded, so that a nut would fix a disc, for instance, in position for rotation. The milled nut is fixed on this screw, and a piece of cardboard held against the projections. As the wheel is turned slowly a number of taps is heard, but if the speed be increased these become too rapid to distinguish, and a note is heard which remains the same provided that the wheel is revolved at a uniform speed.

If the speed of rotation is increased the note gets higher, or, in musical language, rises in pitch. If the speed be again made constant, but greater than before, we again get a note which

remains the same, but is higher than before. So that what is called pitch, or commonly the height of a note, depends in the case above on the speed at which the milled disc turns. But obviously this means that it depends on the rapidity with which the taps of the card are made on the wheel. When the taps follow each other

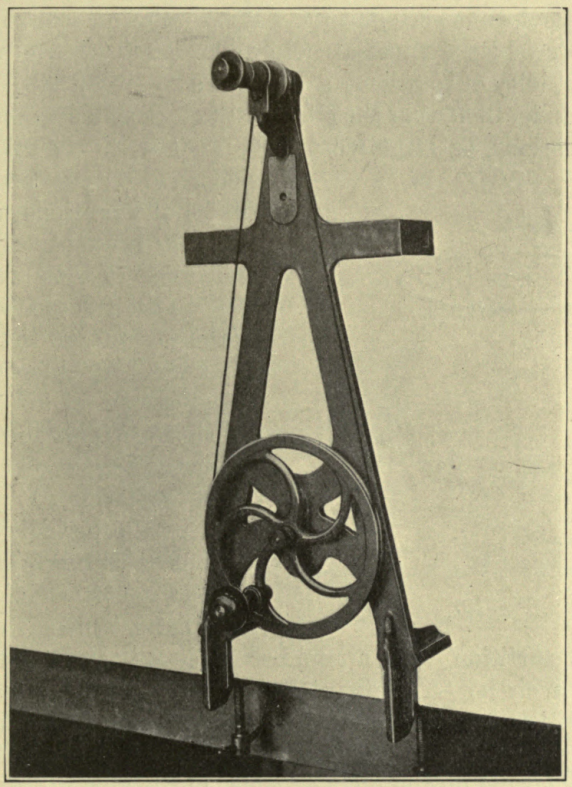


Fig. 9.

slowly we distinguish them separately, but as the speed increases we cannot do so, and their combined effect produces a note.

We can arrive at the same result in another manner. A circular disc of cardboard about 10 ins. in diameter has twenty-four holes, each about $\frac{3}{16}$ in. diameter, punched at equal distances round a circle concentric with its boundary, but whose radius is about $\frac{3}{4}$ in. less. This disc is fixed on the whirling table, and just above the

circle in which are the holes is placed a glass jet connected to a gas-bag filled with air (*see* Fig. 10). If a hole happens to be opposite the jet, air is blown through it, and we hear a sharp puff; but if a piece of the card faces the opening, there is no sharp puff, but we hear a more gentle sound, due to air escaping less rapidly. If now the disc is rotated slowly, a succession of puffs can be heard, but exactly as in the previous experiment more rapid rotation causes them to blend and produce a musical note. The pitch of the note rises with the rapidity of rotation just as before.

We conclude, then, that the pitch of a note depends on the rapidity of the vibrations, or impulses, which produce it. In other words, it depends on the frequency or number of vibrations per second.

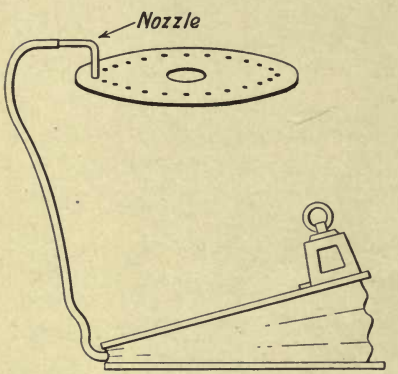


Fig. 10.

In both these experiments we could alter the loudness of the note—in the first case by increasing the pressure of the card; in the second, by increasing the rate of escape of air from the gas-bag by loading the bag with additional weights.

The method of producing sound by the periodic escape of air from holes has been utilised in the form of a whistle. Inside the whistle, which is a circular tube, is a partition with a number of holes cut in the form of sectors of a circle. A similar plate with corresponding slits, the edges of which are turned up like the vanes of a ventilating fan, can revolve over the first plate about an axis through its centre. The wind blowing on the inclined edges of the slits causes the plate to revolve, and so periodically cuts the wind off. The result is a sound which rises or falls in pitch as the plate increases or decreases in speed.

§ 15. *The Siren.*—An adaptation of the rotating disc already described can be used to determine the frequency of a given note and is called a siren. The disc is of metal, and instead of there being one jet, there is a number of jets, arranged so that all the holes in the disc are opposite all the jets at the same time. Thus a more intense puff of air is obtained. The disc has several

circles of holes, in order to allow a greater range of frequency, and is revolved by a motor which can be regulated to give different speeds. If we can determine the speed at which the disc is revolving we can find the frequency of the note produced. For if there are twenty-four holes in the disc there will be twenty-four puffs per revolution, because each time a hole is opposite to a jet there is a puff, and all the jets are opposite to holes at the same time. To know the number of vibrations made per second, which is the same as the number of puffs per second, we only require to know the number of revolutions of the disc per second.

This is found by means of a counting apparatus and a stop-watch. The principle of the counter is as follows: the axis of the disc has a thread turned on it (*see* Fig. 11), and this thread engages a toothed wheel. Each revolution of the axis gives the screw a complete turn and makes one tooth of the wheel move on to the place of the next. So if there are 100 teeth on the wheel it will make a complete revolution when the disc has made 100 revolutions.

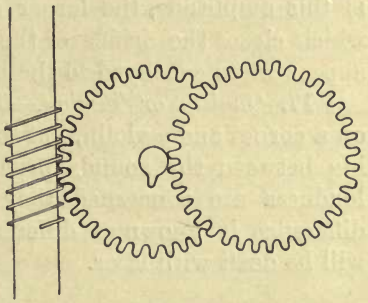


Fig. 11.

A hand attached to the axis of this wheel can be made to revolve with it round a dial and so indicate its revolutions.

On the axle of this first toothed wheel is a single tooth which catches the teeth of a second toothed wheel once in every revolution of the first. This works in the same way as the simple cyclometers attached to bicycles.

If the second wheel has 100 teeth, it will make one revolution for every 100 of the first, that is, for every 10,000 of the disc. The dial belonging to the second wheel would therefore be divided into ten parts each indicating 1,000 revolutions. We can now count the revolutions, and by seeing how many take place in a given time we can find the number of vibrations which produce the note being given out by the siren.

In using the siren to determine the frequency of a note given out by a tuning fork (say), we arrange the speed of the motor so that the siren gives out the same note, and then count its vibrations as above.

We now know that difference of pitch is due to difference in the number of vibrations per second producing a sound, and it occurs to us that perhaps the velocity of sound might vary for notes of different frequencies. If this were so, we should be unable to recognise the tune played by a band at a distance, for the time would get all awry, and some notes might reach us before those which originally preceded them. None of these things have been detected, consequently notes of different frequencies travel at the same pace.

§ 16. *Intensity.*—Two notes may have the same pitch, or frequency of vibration, but may differ in intensity, or loudness. We have already explained that the loudness of a sound merely depends upon the amplitude of vibration of the air particles. The greater is this amplitude, the larger are the vibrations of the membrane which closes the orifice of the ear, and the louder does the sound appear when conveyed to the brain by the mechanism of the ear.

§ 17. *Quality or Timbre.*—The same melody may be performed on a cornet and a violin, but the ear has no difficulty in distinguishing between the sounds produced, although as far as pitch and loudness are concerned they may be exactly the same. This difference is known as difference of quality or timbre; its causes will be dealt with later.

*Examples.*

1. Two sounds differ in pitch: to what physical characteristic is this difference due? How would you compare their pitches? (Oxf. Prelim. and C. (3) 05.)

2. How may a siren be used to find the pitch of a tuning fork?

If the movable disc of a siren is perforated by 15 holes and makes 600 revolutions per minute, find the pitch and wave-length of the note emitted, assuming the velocity of sound to be 340 metres per second. (Army 04.)

3. On what physical quantities do (1) the pitch; (2) the "quality"; and (3) the loudness of a musical note depend? (Army 01.)

4. Describe how to determine the pitch of a tuning fork by means of a siren. If you were given a number of tuning forks differing slightly in pitch, the vibration number of one of them being known, how could you determine the vibration numbers of the rest? (Camb. Loc. Sen. 07.)

5. A man cracks a whip near a line of vertical wooden palings, and after each crack a shrill note is heard. Explain this. (Matric. 04.)

## CHAPTER IV

### NOTES USED IN MUSIC

§ 18.—It is well known that only certain collections of notes produce pleasing effects. Thus, a piano which is out of tune produces a very unpleasant effect when played, for the instrument when in tune was enabled to produce notes which agreed with each other, so to speak, but when out of tune all these notes have changed slightly, most of them having been lowered in pitch or flattened, and the result is disagreeable. The same thing occurs in the case of a violin played by a poor performer.

It is found that those notes which produce agreeable sounds, when played together or in succession, have frequencies the ratios of which can be represented by small numbers, and that the smaller are the numbers forming the ratio, the more pleasing is the effect produced when the two notes are sounded together. Thus a note and its octave when sounded together are perfectly in accord, and it is found by means of the siren that the frequency of the octave is twice that of the fundamental, whatever be the numbers representing the frequencies. Between the fundamental and its octave, for musical purposes a number of other notes are placed. There are seven principal notes whose frequencies bear simple ratios to that of the fundamental. These ratios are given below, the notes being numbered from 1 to 8.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | $\frac{9}{8}$ | $\frac{5}{4}$ | $\frac{4}{3}$ | $\frac{3}{2}$ | $\frac{5}{3}$ | $\frac{15}{8}$ | $\frac{2}{1}$ |

The names

| *doh* | *ray* | *me* | *fah* | *soh* | *lah* | *te* | *doh* |
|---|---|---|---|---|---|---|---|
| $\frac{9}{8}$ | $\frac{10}{9}$ | $\frac{16}{15}$ | $\frac{9}{8}$ | $\frac{10}{9}$ | $\frac{9}{8}$ | $\frac{16}{15}$ | |

are sometimes given to the notes of the scale, and the ratios of the frequencies of each note to that of the preceding one are given

below these names. The following numbers enable one to remember the relative frequencies of the notes of the scale:—

24 27 30 32 36 40 45 48

all of which contain only 2, 3, 5, as prime factors.

Beginning again with the octave the same sequence of relative frequencies can be constructed, all the notes of which will be the octaves of those in the first series, and so on. On a piano there are generally seven octaves, and since the first octave has double the frequency of the lowest note, the second double this, and so on, the highest note has a frequency $2^7$ times that of the lowest.

The scale given could be constructed with any number for its fundamental frequency, provided that the frequency corresponds to an audible sound, for, as we have seen, if the vibrations are too slow they are not heard as a note. Similarly, too great a frequency does not produce an audible sound. It is usual to adopt a standard pitch, and fixed names for the notes which will then occur. For this the letters $a$ to $g$ are used, with dashes or suffixes to indicate octaves above or below the middle one—for instance, $c$, $d$, $e$, $f$, $g$, $a$, $b$, $c'$, represent the middle octave on the piano, and $a$ in this octave is used for a standard of reference. In England concert pitch is 460 vibrations per second for $a$, in France 435, and in Germany 400. But a more convenient standard for physical purposes is to take $c$ as 256, which is a power of 2.

For musical purposes the notes given above do not allow sufficient variation, so additional notes are introduced between some of these, and are called sharps and flats. Then, too, the scale beginning on $c$ might not be a convenient pitch for a singer, while raising or lowering the pitch by an octave would be too great a variation, so it is necessary to have scales beginning on the other notes, $d$, $e$, $f$, etc.

Let us see what we get by introducing notes between those already given.

We have—

| $c$ | | $d$ | | $e$ | $f$ | | $g$ | | $a$ | | $b$ | $c'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\frac{9}{8}$ | | $\frac{10}{9}$ | $\frac{16}{15}$ | | $\frac{9}{8}$ | | $\frac{10}{9}$ | | $\frac{9}{8}$ | $\frac{16}{15}$ | |

the smallest intervals of which are those between $e$ and $f$ and $b$ and $c'$ where the ratio is $\frac{16}{15}$. This interval is called a semi-

tone, and the larger intervals $\frac{9}{8}$ or $\frac{10}{9}$ are called tones. If we introduce notes in the larger intervals differing by semitones from the notes already given we shall get between *c* and *d*, for instance, *c* sharp, written *c*♯, with a frequency of $256 \times \frac{16}{15}$, or 273, approximately, and *d* flat, written *db*, with a frequency of $\left(256 \times \frac{9}{8}\right) \times \frac{15}{16}$, or 270, so that c♯ and *db* would be slightly different notes.

In the same way, starting with *d* as a fundamental or keynote for a scale, the next principal note would not be *e*, because the interval between *d* and *e* is $\frac{10}{9}$, while that between *doh* and *ray* should be $\frac{9}{8}$.

On an instrument like the violin all these notes can be obtained, but on a keyboard instrument like the piano it is impossible on account of the number of keys required.

But we have seen that the difference between *c*♯ and *db* is small, and so is the second difference mentioned. To overcome the difficulty of the number of keys, what is called a tempered scale is adopted, and in it the two kinds of tone are equalised and made equivalent to two semitones. This being so, *c*♯ and *db* will be the same note, and since all tones are equal the same set of notes will do for scales with any keynotes. Only the octaves are absolutely correct, but the errors are so small as to be absolutely inappreciable. We have now twelve equal intervals between a note and its octave. So that if $x$ is the fraction which expresses this interval, and the keynote is *c* (say) whose frequency is 256, *c*♯ or *db* has a frequency $256 \times x$, *d* has a frequency $(256 \times x) \times x$, *d*♯ or *e*♭ $256 \times x^3$, *e* $256 \times x^4$, and so on until *c'* has a frequency $256 \times x^{12}$.

But we know the frequency of *c'* is $256 \times 2$.

$\therefore x^{12} = 2$, or $x = \sqrt[12]{2}$, which is 1·059 approximately.

Below are given the ratios of the frequencies to those of the keynote in the first or natural scale and in the tempered scale :—

| *doh* | *ray* | *me* | *fah* | *soh* | *lah* | *te* | *doh* |
|---|---|---|---|---|---|---|---|
| 1 | 1·125 | 1·25 | 1·333 | 1·5 | 1·667 | 1·875 | 2 |
| 1 | 1·122 | 1·26 | 1·325 | 1·498 | 1·682 | 1·888 | 2 |

§ 19. *Beats.*—We have remarked that certain notes when sounded together produce pleasing effects, and other combinations produce unpleasant effects. If two notes differing slightly in frequency be sounded together a periodic increase and decrease of loudness can be heard and a throbbing sensation is produced. To this phenomenon the name of "beats" is given. Very slow or very rapid beats are not distasteful, but there is a number which gives a

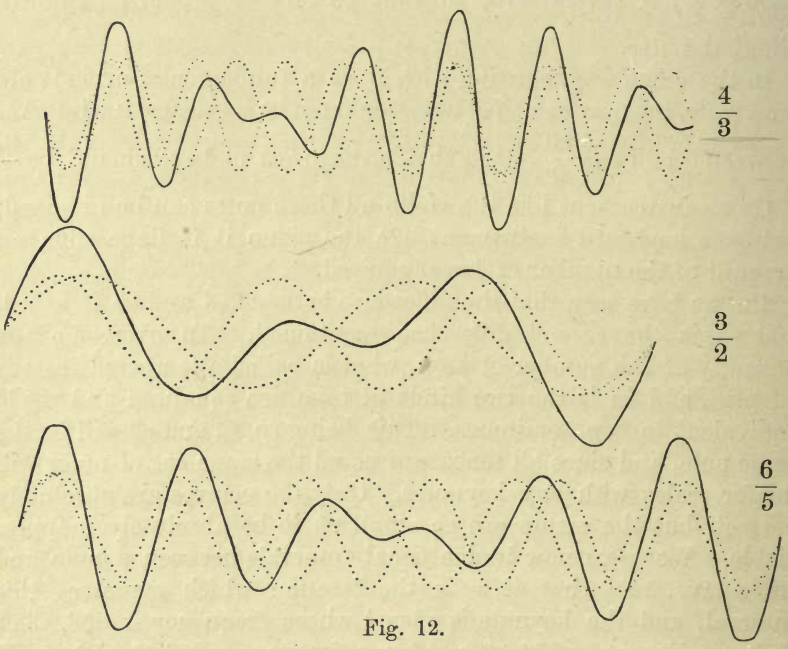



Fig. 12.

maximum of unpleasantness, and this number varies with the pitch of the notes producing the beats. The cause of beats can easily be shown by means of a diagram, as in Fig. 12.

When sound passes any point the pressure is alternately slightly greater and slightly less than the normal pressure. In the diagram the ordinates represent excess of pressure when above the axis, and deficiency when below. Thus the two dotted lines in the upper part of the figure represent the changes of pressure which would take place if waves of sound whose frequencies were in the ratio of 4 : 3 passed separately. The continuous line represents the effect of both sets passing together.

We see that when compressions in both waves are at any point together there is a great excess of pressure at that point. Later on a compression is passing in one set while a rarefaction passes in the other. The total result is that a series of big compressions and rarefactions is followed by a series of small ones. But the loudness of a sound depends on the magnitude of its compressions and rarefactions, or, in other words, on the amplitude of motion of the air particles. Hence the sound is alternately louder and softer.

As an example, let us find the number of beats per second given by two notes of frequencies 80 and 84. The second note makes 21 vibrations, while the other makes 20. Therefore, if they started so that compressions of both reached the ear together, in 21 vibrations of one and 20 vibrations of the other, they would both be together again. This would happen four times a second, so that there would be four alternations of sound or four beats per second. And, in general, the number of beats per second is given by the difference of frequencies.

*Examples.*

1. Explain the terms *period of vibration*, *wave-length*, and *velocity*, as applied to a sound-wave.

If a tuning fork of frequency 256 is sounded in air in which the velocity of sound is 340 metres per second, what is the wave-length of the note produced by the fork? (Army 02.)

2. Explain the cause of beats. How would you find the vibration number of a given tuning fork if you were provided with a second fork of nearly the same pitch and of known vibration number? (Army 02.)

3. Describe experiments to show that the impression of a musical interval as judged by the ear depends solely upon the ratio of the frequencies of vibration of the two notes concerned and not upon the difference of their frequencies.

The frequencies of vibration of two notes being 400 and 900, what is the frequency of a note that would appear to the ear to lie midway between them? (Matric. 08.)

4. Explain the formation of the beats heard when two tuning forks which are not quite in unison are sounded together. A standard fork $A$ has a frequency of 256 complete vibrations per second and, when a fork $B$ is sounded with $A$, there are four beats per second. What further observation is required for determining the frequency of $B$? (Camb. Loc. Sen. 06.)

5. Does the number of beats per second between two notes which form a definite musical interval depend on the relative or on the absolute pitch of the notes? (Camb. Loc. Sen. 07.)

## CHAPTER V

### VIBRATING SOLIDS

§ 20. *Tuning Forks.*—We have already mentioned that a bar clamped in a vice by one end when vibrating gives out a musical note. Any vibrating solid will produce audible sound provided that its vibrations are not too slow nor too rapid. An instrument called a tuning fork is a convenient means of producing a musical note. Such a fork is shown in Fig. 13. Forks are generally made of steel, and, provided that no alteration in size or shape takes place, any particular fork will always vibrate at the same rate, that is, will always produce the same note. The way in which it vibrates is shown in Fig. 13.

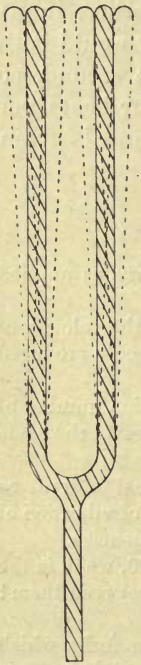


Fig. 13.

§ 21. *Determination of the Frequency of a Fork.*—The following is a rough method of determining the frequency of a fork. The apparatus used is shown in Fig. 14. A is a pendulum, with a stout bristle affixed to the bob by a little wax. B is the fork, which is screwed into a heavy block of wood, and to one of its prongs another bristle is attached. The fork and pendulum are adjusted so that the bristles just touch a plate of glass C, on which soot has been deposited by holding the glass over the flame of a paraffin lamp. Careful adjustment is needed so that the bristles just touch the plate lightly and do not impede the vibrations at all. A piece of brass wire filed to a fine point works better than a bristle, but is more difficult to adjust. The pendulum is now set swinging across the glass perpendicularly to its length. One edge of the glass rests against a ledge of wood, which acts as a guide. As long as the glass is not moved, the pendulum traces out the same straight line. It can be best started by pulling the bob on one side and fixing it with a thread. If the thread be

burned, the bob starts to swing in a straight line. If the fork be made to vibrate by drawing a well-resined violin bow across the prongs, the bristle attached traces out another straight line. On moving the plate of glass along the table the bristles make traces

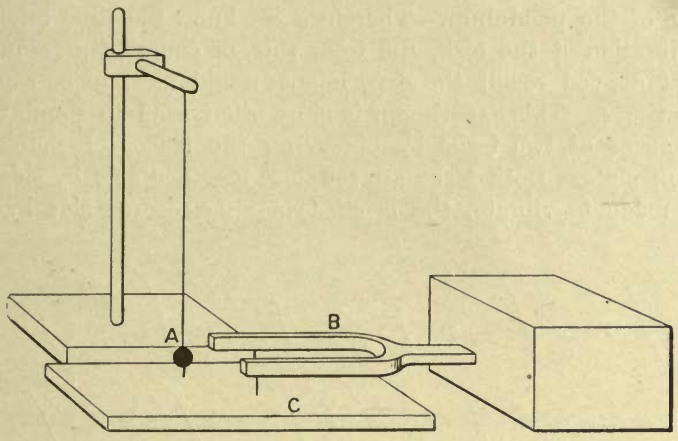



Fig. 14.

like those shown in Fig. 15. That made by the fork is a continuous wavy line, while a similar trace would be made by the pendulum bristle if it touched the glass all the time. As it is, portions of this trace only are made, and the line made by the fork is crossed at intervals by the trace of the pendulum. In making an experiment the pendulum and fork can be set in motion, and the plate moved as uniformly as possible by hand. The two

Fig. 15.

bristles should be set fairly close together, and then from the trace we can count the number of vibrations made by the fork during one swing of the pendulum. Suppose the trace is like that of Fig. 15, A and B being the traces of the pendulum made at successive crossings of the plate—*i.e.*, the traces are made at intervals of half a vibration. The vibrations of the fork are marked off at

$c\ c'\ c''$. There are in our figure $3\frac{1}{2}$ vibrations of the fork to one half vibration of the pendulum. In practice the fork would make a larger number of vibrations during one of the pendulum. By timing the pendulum with a stop-watch, we can find the time of an oscillation, and we know the number of vibrations of the fork to one of the pendulum. Therefore we know the time taken for one vibration of the fork, and from this, of course, the frequency. To get a good result the experiment would need to be repeated many times. This experiment is more often used to compare the frequencies of two forks by observing the traces of both on a moving plate. Both forks are set close together side by side, and then made to vibrate, the bristles making records side by side.

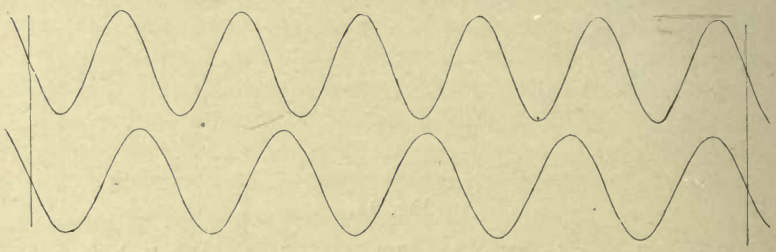


Fig. 16.

The traces made by two forks whose frequencies were in the ratio 6 : 5 are shown at Fig. 16.

§ 22. *Vibration of Strings.*—We shall consider a string to be a thin uniform flexible thread stretched between two points. In order to study the vibrations of strings experimentally, an instrument called a sonometer* (Fig. 17) is employed. This consists of a long rectangular wooden box, at each end of which are pegs for stretching the strings. The strings pass over bridges or wedges of hard wood tipped with metal, so that we can study the vibration of a definite length of string intercepted between two bridges. A stretched string can vibrate in two ways. It can either move perpendicularly to its length, like the clamped bar which has already been mentioned (or like the prongs of a tuning fork); it is then said to be vibrating transversely; or the various portions can move in the direction of its length as the particles do when sound travels in air. Such vibrations are longitudinal. Transverse vibrations are the only ones with which we shall deal.

* Or monochord.

§ 23. *Experiments with the Sonometer.*—A piece of piano wire of medium thickness is fixed on the sonometer, and is stretched by turning the pegs to which it is attached. The movable bridge is placed at one end so as to allow as long a string as possible to be used.

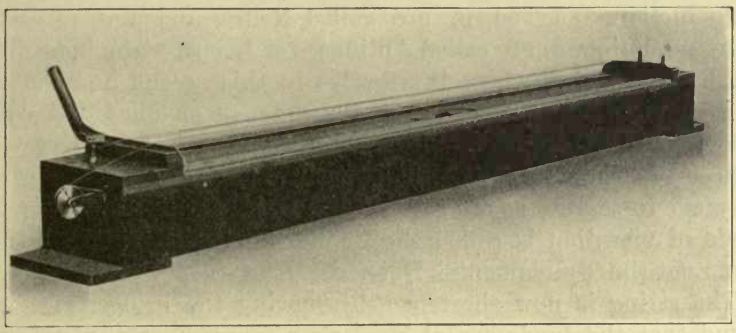


Fig. 17.

By plucking the string or drawing a violin bow across it, it can be made to vibrate and give out a note. That it is vibrating is easily proved either by touching it with the finger or by placing small riders of paper on it. Some of these will be thrown off. These riders

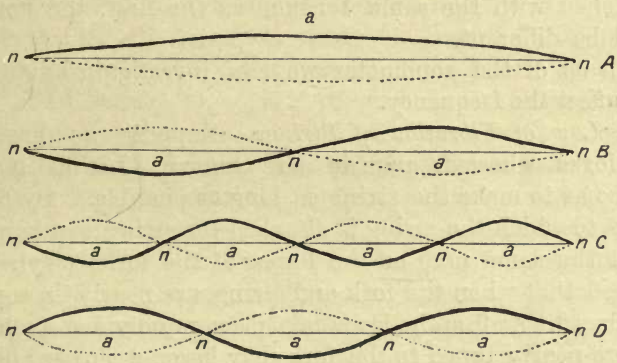



Fig. 18.

afford a means of examining the motion of the string. For instance, if the string be strongly plucked or bowed in the middle, and riders be placed along the string, the motion will be found to be most violent in the middle, decreasing gradually towards the ends. The string is therefore vibrating as in A (Fig. 18). If the vibration is strong enough the string may be seen apparently to spread out into a shadowy spindle.

Now grasp the wire at the middle between the thumb and finger. This will have the effect of stopping the motion at that place. By bowing at a point a quarter of its length along the string it is seen to vibrate as at B, or its motion can be examined by riders as before. The places of no motion, or in reality very little motion, marked *n*, are called Nodes, and the places of greatest motion, *a*, are called Antinodes or Loops. The note given out by the string when it vibrates in this second method will be recognised as the octave of the note given out in the first mode, so that the rate of vibration is twice as fast in the second case as in the first. It will be easily seen how the string can be made to vibrate as in Figs. C and D. The first and simplest mode of vibration is called the Fundamental Mode, and the note given out the Fundamental Note.

The string is now shortened by moving the bridge, and it is found that the fundamental note rises in pitch as the string is shortened. The pitch also rises if the string is tightened—that is, if its tension is increased. This fact is applied in tuning a violin or piano. If a thicker string be substituted and the length and tension be the same, the note produced is lower. Similarly, if a wire of the same length and thickness but of different material be stretched with the same tension as the first, the note given out will be different.

By means of the sonometer we can investigate how all these factors affect the frequency.

§ 24. *Law of Vibration of Strings.*—Suppose we have three tuning forks whose frequencies are known. Let us move the bridge so as to make the string as long as possible. By means of the pegs to which the string is attached the string can be made to give out the same note as the lowest of the forks (say). It will be noticed that when the fork and string are nearly in tune beats are produced. At first the beats take place rapidly, but as the string gets more nearly tuned to the fork they become slower and slower and, when perfect unison is obtained, disappear. The length of string between the bridges is measured, and the string is then shortened until in tune with another fork, when it is again measured. This is repeated with the third fork. Care must be taken that the string vibrates each time in its fundamental mode. It will be found that the lengths of string required to give notes of these three different frequencies will be inversely proportional to

the frequencies. In symbols, $n \propto \frac{1}{l}$ where $n$ is the frequency, and $l$ the length of the string. When the string vibrates with nodes other than those at the ends, we can consider that it has become divided into smaller strings, each portion between two nodes being a separate string.

To find the effect of change of tension on frequency, instead of fixing both ends of the wire to pegs on the sonometer, let one end pass over a pulley and support a light pan or hook on which weights can be placed. We can then tune the string to the different forks by hanging different weights on it. It will be found that the frequency of the fundamental note varies as the square root of the stretching weight. For example, if with 14 lbs. we get a certain note, with 56 lbs. we shall get a note of twice the frequency, or, in other words, the octave.

We shall investigate the effects of changes of thickness and material of strings by using one string whose length can be altered at will, and comparing with it other strings the lengths of which are the same and which are stretched by the same force.

First of all, let us take samples of the various strings and weigh them. By measuring the lengths of these, we can find the mass of 1 cm. of each of the strings. Each string is now used in turn and the length of the fixed string is altered until it gives out the same note as the other string. By measuring the lengths of the fixed string which are in tune with the others we can find the ratios of the frequencies of these, for the frequency of the note of the fixed string is inversely proportional to its length. So that if the lengths of the fixed string are 160 and 100 cms., the frequencies are proportional to $\frac{1}{160}$ and $\frac{1}{100}$; that is to say, the frequency of the note of the first is $\frac{5}{8}$ of the frequency of that of the second. We shall find that the frequencies of the different strings are inversely proportional to the square roots of the masses of 1 cm. of each.

Suppose one string weighs 0·4 gram per cm. and another 0·1, the note given out by the second will be higher than that of the first, and it will be the octave, its frequency being twice as great.

A series of experiments with wires of different materials and thicknesses will show that the frequency is always inversely proportional to the square root of the mass per unit length, provided that the length and tension do not vary.

We can sum up these results in the formula, frequency $\propto \frac{1}{l}\sqrt{\frac{T}{\rho}}$ where $l$ is the length, $T$ the tension, and $\rho$ the mass per unit length. If $l$ is in centimetres, $T$ in dynes, $\rho$ in grams per centimetre, the frequency is given exactly by

$$n = \frac{1}{2l}\sqrt{\frac{T}{\rho}}.$$

That is, in the case of a string 100 cms. long, having a mass of 0·1 gram per centimetre, stretched by a tension of 240 kilograms weight, the frequency is

$$n = \frac{1}{2 \times 100}\sqrt{\frac{240{,}000 \times 981}{\cdot 1}} = 256 \text{ approximately.}$$

The factor 981 is necessary to convert grams weight to dynes (*see* § 13, D.).

§ 25. *Melde's Experiment.*—An interesting way of showing the vibrations of a string is the following:

A string is attached to one prong of a tuning fork, and the other end of the string passes over a pulley and supports a scale pan on which weights can be placed (*see* Fig. 19). In the diagram the size of the fork is exaggerated. If the arrangement is like that given, when the prong of the fork moves towards A, the string is loosened and sags; as the prong moves back it is tightened again and is straight when the prong is at C; then as the prong moves back towards A the string is carried on by the upward motion it has acquired, and so vibrates up and down. While the fork moves from C, when the string is straight to A, the string moves up to *a*, and as the fork moves back to C again the string moves from *a* to *c*. So that for one vibration of the fork the string makes half a vibration. Hence its frequency is half that of the fork. For the string to vibrate strongly in its fundamental mode it is necessary to have a certain definite tension. Let us suppose that this was exactly obtained in the preceding experiment.

Now let the tension be gradually reduced by removing weights from the pan. The string first of all ceases to vibrate strongly, but when a certain tension is reached it again vibrates, but with a node in the middle and two vibrating segments. By further decreasing the tension we can get three, four, and more segments.

The reason for this can be seen from the formula

$$n = \frac{1}{2l}\sqrt{\frac{T}{\rho}}.$$

$n$ is the frequency of the string, which is half that of the fork, and remains constant when the string vibrates. If we decrease $T$, $l$ must also decrease for $n$ to remain constant, but $l$ can only

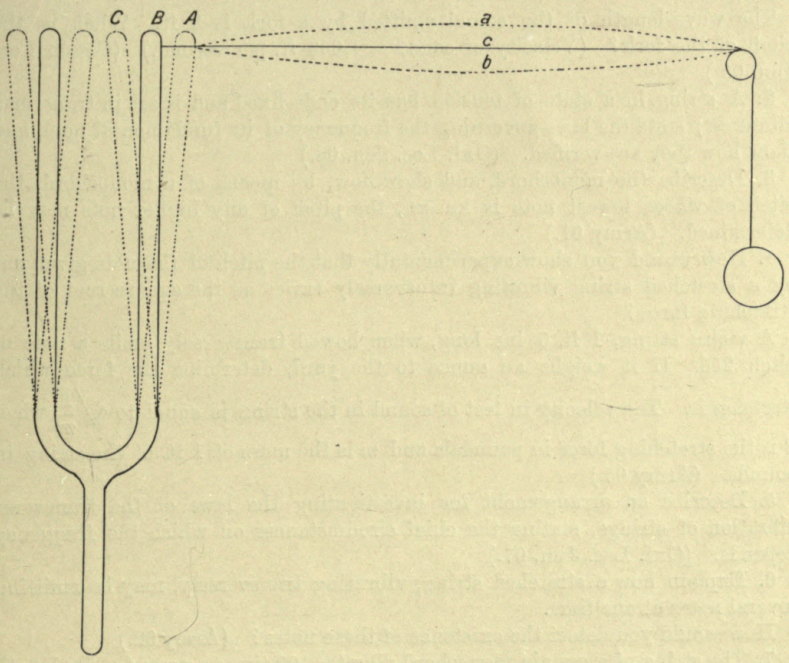



Fig. 19.

decrease by the string vibrating in segments, each of which for determining frequency must be taken as a separate string. For the string to vibrate in one, two, three or more segments, there are well-defined values of $T$, and for (intermediate) values of $T$ the string does not vibrate.

The experiment can be performed roughly as follows: A piece of silk fishing line several yards long was attached to a fork whose frequency was 50. The other end was held by an observer who could adjust the tension sufficiently well to make the string vibrate with from one to six segments.

If the fork be placed so that its movement is perpendicular to the direction of the string, the string will vibrate with the same frequency as the fork, for they will both be at the extreme points of their motion together.

*Examples.*

1. How would you determine the wave-length of the sound emitted by a tuning fork?

The wave-length of the sound emitted by a fork is 3 ft. What is the pitch of the fork? (Velocity of sound = 1,093 ft. per second.) (Camb. Loc. Jun. 05.)

2. A string in a state of tension has its ends fixed and is set in transverse vibration; state the laws governing the frequency of its fundamental note, and show how they are verified. (Oxf. Loc. Sen. 08.)

3. Describe the monochord, and show how, by means of a monochord, the pitch of whose lowest note is known, the pitch of any higher note may be determined. (Army 01.)

4. How would you show experimentally that the pitch of the note given out by a stretched string vibrating transversely varies as the square root of the stretching force?

A violin string, 1 ft. 6 ins. long, when bowed transversely emits a note of pitch 256. If it weighs an ounce to the yard, determine the force which stretches it. The velocity in feet of sound in the string is equal to $\sqrt{\frac{T}{m}}$ when T is the stretching force in poundals and $m$ is the mass of 1 ft. of the string in pounds. (Army 05.)

5. Describe an arrangement for investigating the laws of the transverse vibration of strings, stating the chief circumstances on which the frequency depends. (Oxf. Loc. Jun. 07.)

6. Explain how a stretched string, vibrating transversely, may be emitting several notes at one time.

How would you detect the existence of these notes? (Army 02.)

7. The string of a certain monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the relation of the new tension to the original? (Army 02.)

8. Describe any method of counting the number of vibrations made by a tuning fork. In what way would you expect temperature to affect the number, and why? (Lond. Inter. Sci. 05.)

9. Describe an experiment to determine the ratio of the lengths of two similar strings stretched with the same tension when the interval between the notes sounded by them is an octave. (Camb. Loc. Sen. 07.)

10. What is a musical interval? How would you determine the lengths of a stretched string corresponding to some definite musical interval? (Camb. Loc. Sen.)

11. Describe a monochord and explain how to use it to compare the frequencies of two tuning forks.

If the frequency of the middle C on a pianoforte be 256 vibrations per second, what will be the frequency of the next higher E? (Camb. Loc. Sen. 06.)

12. Compare the frequencies of vibration of two strings stretched with weights of 10 kilograms and 1 kilogram respectively. They are 1 metre long each, and they are of the same diameter, but their densities are 7·8 and 1 respectively. (Lond. Inter. Sci. 02.)

## CHAPTER VI

### VIBRATIONS OF COLUMNS OF GAS

§ 26.—A column of air enclosed in a tube, closed or open at its ends, can be made to vibrate and give out a sound. For instance, if a tube (Fig. 20) about 2 ins. wide be placed upright in water so as to stop one end and the length of the air column be suitably adjusted, when a tuning fork is placed over the open end the sound of the fork is reinforced. If the length of the tube is properly chosen, the sound will be heard even though that given by the fork alone is inaudible. The vibrations of the fork throw the air in the pipe into vibration and an augmented sound is produced. The vibrations of air columns are analogous to those of strings. There is this difference, however. The particles of a string move perpendicularly to its length, but the air particles move to and fro in the direction of the axis of the pipe.

Let us see what happens in the case we have mentioned. It will be found that a slight alteration in the length of the air column causes the reinforcement of sound to cease. That is, there is a definite length of air column to give a certain note. With a fork producing a different note, it will be found that the tube has to be raised or lowered in the water. Suppose instead of the prong of the fork we had a piston fitting the tube loosely and moving in exactly the same way as the prong moves. Let the piston move from B, in Fig. 20, to C. This causes a compression of the air at the top of the tube which travels down it with a certain velocity which is the same for all compressions or rarefactions in air, and is the velocity of sound. When this compression reaches the water it will be reflected and travel up the tube again. In a compression the particles are moving forward, so if the compression, which started when the piston was at B, travels down and up the tube while the piston moves to C, the air particles near the mouth are now ready to move with the piston as a consequence of the compression, and so the motion of the piston is assisted. The upward motion of the piston now

causes a rarefaction which travels with the same velocity as the compression, and therefore moves down and, after reflexion, up the tube again while the piston is reaching B. The piston and the air are now in the same state as they were initially, and the motion is repeated. For the motion to be repeated it is necessary for a compression to traverse the tube in such a time that it arrives at the top when the fork is at C. This may happen if the compression goes down and up while the fork makes half a vibration, or while it makes one and a half vibrations, and so on. In the second case there will be more than one compression travelling along the tube at the same time, and two compressions will pass each other. When this takes place, the air at the place where the compressions pass is doubly compressed. The effects of the compressions are added together. In the same way two waves meeting each other on the surface of water pass on without alteration, but at the moment of passing the waves are added together, so to speak, and a higher crest is formed.

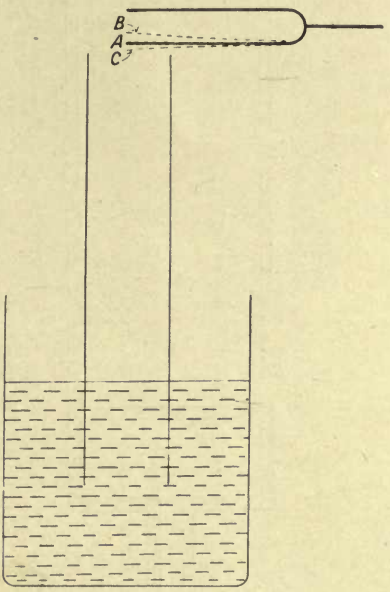


Fig. 20.

In the case in which the compression travels down and up the tube while the piston makes half a vibration, the air in the tube goes through all its changes while the piston makes a complete vibration, for it is traversed by a compression and a rarefaction. The air then goes through its changes in the same time as the piston, and therefore makes the same number of vibrations per second. Thus the note given out by the air column would be of the same pitch as that given out by the fork, supposing the piston to vibrate at the same rate as the fork. When the fork replaces the piston the effect is the same, but the compressions are not so intense, and therefore the sound is weaker.

A compression travels down and up the tube during half a

vibration, and therefore travels a distance equal to four times the length of the tube in the time of a complete vibration.

Therefore (*see* Chapter I.) the length of the tube is a quarter of a wave-length. At the end of the tube stopped by the water there is the least motion of the air particles. In fact there can be no

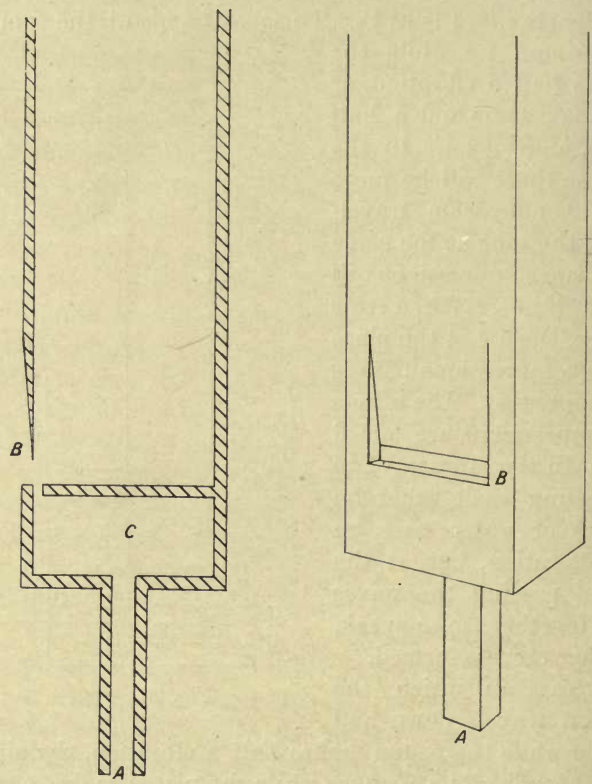



Fig. 21.

motion perpendicular to the surface in the layer next the water. On the other hand, the greatest motion of the air particles takes place at the open end of the tube. By analogy with the case of a vibrating string, places of no motion are called nodes, and places of maximum motion antinodes, or loops, or ventral segments. Although at the closed end of the tube there is very little motion of the particles, yet the pressure there changes, for there are alternately compressions and rarefactions being reflected. At the

open end of the tube the changes of pressure cannot be very great. This has been verified by experiment.

§ 27. *Organ Pipes.*—To produce a note the air in an organ pipe is caused to vibrate by means of one of two devices, which will be described later. The mode of vibration of a stopped pipe is exactly

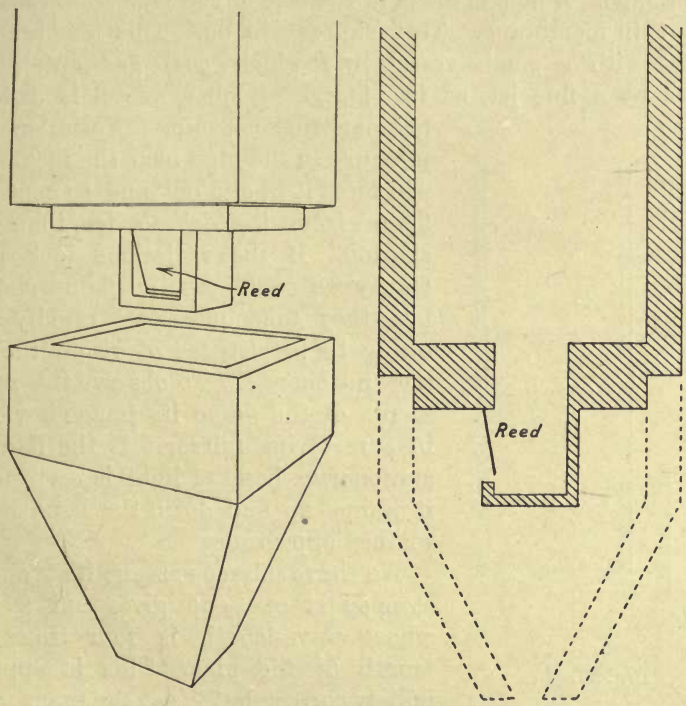



Fig. 22.

the same as that of the column in the last experiment. Ordinarily it vibrates having a node at the closed end and a loop at the open end.

One method of causing the vibration is practically the same as that employed when sound is produced by blowing across the mouth of a key. A section of the open end of a pipe is shown in Fig. 21. Air is forced in through A and, in escaping from the box C across the aperture at B, sets the air in the pipe vibrating. The sound is produced in exactly the same way in flutes and piccolos and in an ordinary whistle.

The other method is by means of a thin strip of metal called a

reed. Sketches of a pipe with a reed are given at Fig. 22. Air is blown in from below and rushes past the reed into the organ pipe, causing the reed to vibrate, and the vibration of the reed causes vibration of the column of air in the pipe.

We shall now describe a method of examining the vibration of air in a pipe. A hole is made in the side of the pipe and is covered with a thin membrane. Thin rubber will do. This membrane is covered with a small vessel into which coal gas enters and escapes by a fine jet, as in Fig. 23. Such a vessel is called a manometric capsule. Variations of pressure in the pipe near the membrane will force it in and out and so cause the flame from the jet to lengthen and shorten. If the variations took place slowly we could see the flame change, but they take place so rapidly that owing to persistence of vision the eye sees no change. To observe the movements of the flame its image is viewed in a revolving mirror. If the flame is stationary a band of light is seen, but if it jumps up and down the band has a toothed appearance.

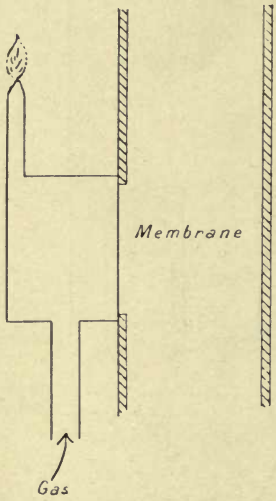


Fig. 23.

We have already seen that a pipe stopped at one end gives out a note whose wave-length is four times the length of the pipe. This is approximately correct, but to get the exact wave-length we must add a correction to the length of the pipe which is proportional to its radius. That the wave-length of the note emitted is approximately four times the length of the column of air vibrating can be easily verified by the experiment described at the commencement of the chapter, if we know the frequency of the fork used and the velocity of sound for

$$v = n\lambda.$$

By lengthening the column of air we come to a place where the tube again resounds. This is the case referred to before, and is due to the fork making one and a half vibrations while a compression travels down and up the tube. In a compression the pressure

is greater than the normal pressure. Let us represent the axis of the tube by a horizontal line AB (Fig. 24), and the excess of pressure at any point of it by the length of an ordinate. Then if the pressure remained normal AB would represent the condition at all points of the tube. But if the pressure was at some places in excess of the normal and below it in others, we should get a curved line going above and below AB.

In Fig. 24 (*a*) a compression is represented as starting down the tube. In (*c*) it has reached the bottom of the tube and a rarefaction has started. In (*d*) the lower dotted line represents the compression travelling down the tube, and the upper dotted line the reflected portion. The two compressions add together to give a resultant excess of pressure indicated by the thick line. In (*f*) the front of the compression has travelled along the tube and back, and the resultant effect is as shown. (*g*), (*h*), (*i*), (*j*) show the condition at later times. We see from these that when the vibrations of the column have completely started, there is no change of pressure at *A* and *D* (these are loops or antinodes), and that the greatest changes of pressure take place at *C* and *B*, the nodes.

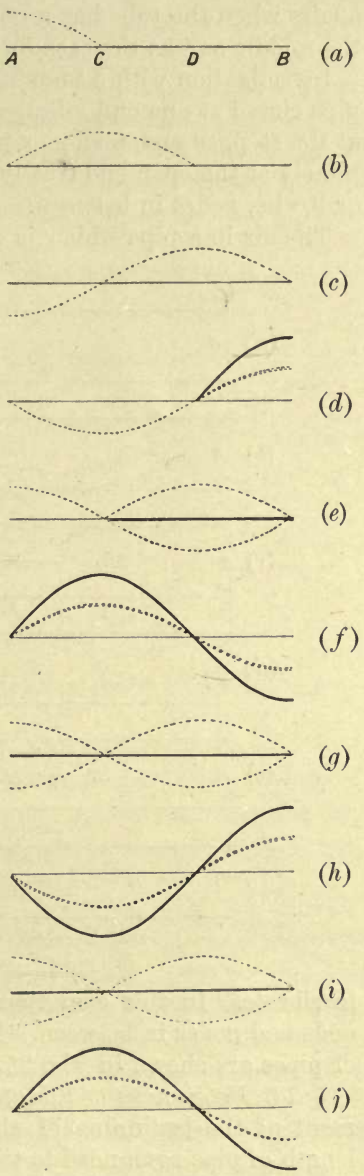



Fig. 24.

The portion of the figure from *A* to *C* represents the state of

affairs when the tube has a node at the closed end and no node in the middle as was the case in the first part of the experiment.

Investigation with a manometric capsule shows that the air in a pipe closed at one end vibrates in the manner shown, with a node at the stopped end, a loop or place of no pressure variation approximately at the open end (really a short distance outside) and 0, 1, 2, or 3, etc., nodes in between.

The air in a pipe which is not stopped at one end can be made

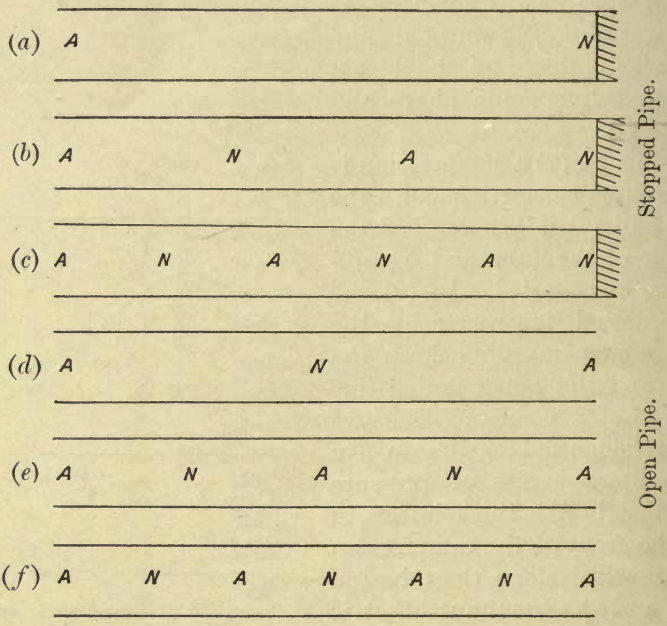



Fig. 25.

to vibrate. In this case there are antinodes or loops at the open ends and nodes in between. The modes of vibration of both kinds of pipes are shown in Fig. 25.

§ 28. *Frequencies of Vibration of Columns of Air.*—In the experiment at the beginning of this chapter we found that a certain length of pipe resounded to the fork, *i.e.*, gave out the same note as the fork. We then found that the same note was produced by a longer column, and had this column been measured it would have been found to be three times as long as the first. The column

was then vibrating as at (*b*) (Fig. 25), but by using a suitable fork it could be made to vibrate as at (*a*) (Fig. 25).

It would then be making fewer vibrations per second. In the first case (*b*), we have seen that one-third the length of the column is one-quarter the wave-length, as the wave-length is four times the distance from node to adjacent antinode. In the second case the length of the pipe is the distance between node and adjacent antinode. Let $l$ be the length of the column.

Then if $\lambda_1$ is the wave-length of the note produced when it vibrates with a node beside the one at the stopped end,

$$\tfrac{1}{3}l = \tfrac{1}{4}\lambda_1 \qquad \lambda_1 = \tfrac{4}{3}l \text{ (approximately).}$$

In the case where there is only one node

$$l = \tfrac{1}{4}\lambda_2 \text{ (approximately)} \quad \lambda_2 = 4l,$$
$$\text{but in the first case } n_1\lambda_1 = v,$$
$$\text{and in the second case } n_2\lambda_2 = v,$$

where $n_1$ $n_2$ are the frequencies.

$$\text{That is } n_1\lambda_1 = n_2\lambda_2, \therefore \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4l}{\frac{4}{3}l}.$$

That is, the frequency of the first note is three times that of the second.

The note given out by a stopped pipe vibrating with one node and one antinode is called the Fundamental, and is the lowest note which the pipe can produce. The higher notes corresponding to 2, 3 and more nodes in the tube, with frequencies of 3, 5, 7, etc., times that of the fundamental, are called Overtones.

In the same way we see that in the case of an open pipe vibrating as at (*d*) (Fig. 25), the length of the pipe is equal to half the wave-length; vibrating as at (*e*) the length is equal to the wave-length, and so the frequencies of the overtones are in the ratios 2, 3, 4, etc., to the frequency of the fundamental.

In general, if a pipe be blown softly the fundamental is produced, but by blowing strongly the first overtone is heard, and sometimes in narrow pipes a strong blast will cause the second overtone to be produced.

§ 29. *Timbre.*—We have before mentioned the difference in quality between sounds of the same pitch, and we shall now explain the cause.

When a pipe is blown softly, we get almost a pure sound, but as the wind blast increases in strength the overtones appear along with the fundamental. It is the presence of overtones which alters the quality of a sound. For instance, the overtones produced in a cornet, although not easily detected, give the notes of that instrument their peculiar distinctive quality.

§ 30. *Resonance.*—The experiment of producing vibration of an air column by means of a fork is an example of resonance. We have explained in that case how the effect is produced. The principle is, however, of very general application, and may be stated as follows :—If a body can vibrate with a certain period, and if vibrations of that period fall upon it, it will commence to vibrate. We can verify this in the case of a string by means of the monochord. A string is tuned to give out a certain note. If, now, that note is sounded near the string, the string will vibrate and throw off paper riders placed on it, and on stopping the sounding body, the string is heard to give out its note.

In the same way, if a company of men are marching across a bridge in step, at such a pace that the interval between their steps coincides with a period of vibration of the bridge, the bridge will vibrate strongly, and may do so to a dangerous extent. It is advisable, then, to break step in crossing a bridge. For the same reason stretcher-bearers do not walk in step.

§ 31. *Resonators and Detection of Overtones.*—A series of pipes could be used to detect overtones, for when the note given by a certain pipe is sounded on another instrument, the air in the pipe will be thrown into vibration. Pipes, however, would have disadvantages as resonators, for a given pipe would resound to its overtones as well as its fundamental. It is necessary, then, to have resonators of such a shape that their overtones will be very different in pitch from their fundamental (very much higher). Then, if used to detect the overtones of an organ pipe, each resonator would, in all probability, only resound to one overtone, for the overtones of the resonator would be so different from the fundamental that pipe and resonator would not have two in common.

The first three overtones of a note on the piano can be easily detected. If the sostenuto pedal of the piano be put down and $C_1$, an octave below middle C, be struck and then stopped by putting a finger on the string, middle C is heard sounding. Similarly G and $C^1$ can be made to sound by striking $C_1$. The explanation is,

that when $C_1$ is struck, C, G and $C^1$ are present also, and if the strings corresponding to these are free to vibrate they will do so in accordance with the principle of resonance.

Other examples of resonance will occur in the section on "Light."

*Examples.*

1. Describe the motion of the air within an open organ pipe when it is sounding its fundamental note.

How could it be found experimentally where, within the pipe, the variation of pressure is greatest? (Army 04.)

2. How would you show the position of the nodes in an open organ pipe sounding its first harmonic? (Oxf. Loc. Jun. 04.)

3. Give a carefully drawn diagram showing the position of the nodes in an organ pipe open at one end and closed at the other for the fundamental vibration and the first three harmonics. (Army 04.)

4. How would you find the frequency of a tuning fork by employing a column of air contained in a cylindrical tube with one end closed?

Draw sketches of the nodes in a pipe open at both ends when it sounds the first two harmonics of its fundamental note. (Oxf. Loc. Sen. 08.)

5. What is meant by the pitch of a musical sound? How can its wave-length in air be determined?

Is the wave-length of a given note different in different media? Give reasons for your answer. (Oxf. Prelim. and C. (3) 04.)

6. If a wire 1 metre long stretched by a force of 10 kilograms vibrates with a frequency of 200, what would be the frequency of vibration of 80 cms. of the same wire stretched by a force of 20 kilograms?

Explain, as fully as you can, the cause of the difference in the character of the sound given out by such a string and of that given out by a tuning fork of the same pitch. (Army 03.)

7. Explain what is meant by resonance and give two illustrations—one mechanical, the other acoustical. (Army 04.)

8. What is meant by resonance?

A source of sound, such as a vibrating tuning fork, is placed at the mouth of a cylindrical tube, and is producing a sound of given frequency; what is the best length of tube for resonance (*a*) if the tube is closed at the other end, (*b*) if it is open? (Oxf. Loc. Jun. 08.)

9. What is meant by the overtones of a note? Find the overtones in the case of an organ pipe open at one end and closed at the other. (Army 01.)

10. Explain, by means of diagrams or otherwise, the motion of the air in sounding organ pipes (*a*) when open at both ends, (*b*) when closed at one end and open at the other. Calculate the frequency of vibration of a closed pipe 11 ft. long. (Velocity of sound in air = 1,100 ft. per second.) (Lond. Inter. Sci. 06.)

11. Describe how you would perform an experiment to find how the pitch of a vibrating column of air depends on the length of the column. (Camb. Loc. Jun. 07.)

12. A glass tube 1 ft. long and 1 in. in diameter is closed at one end. Describe the motion of the air within the tube when vibrating in the simplest possible manner.

How, if at all, would the pitch of the note emitted by such a column be changed by a rise of temperature, by an increase of atmospheric pressure, and by the substitution of a denser gas for the air in the tube? (Matric. 08.)

13. How may two musical sounds differ from each other? and what are the physical phenomena which characterise these differences? Describe how you would investigate the difference in wave-length of two tones. (Oxf. Prelim. and C. (3) 08.)

14. A note on a harmonium and a string of a violin have been tuned to be in unison with a given tuning fork. How do you account for the difference in the quality of the sounds produced by the two instruments? (Camb. Loc. Sen. 06.)

15. What is meant by a fundamental note and its overtones?

Explain how you would experimentally demonstrate the existence of overtones in the case of (*a*) a stretched string vibrating transversely, (*b*) a closed organ pipe.

Would there be any marked difference in the nature of the overtones in the above cases if the fundamentals were the same? (Matric. 08.)

16. A vibrating tuning fork is held near the mouth of a cylindrical tube 32 cms. long closed at one end, and the tube is found to "speak." Explain clearly why this happens.

Assuming the velocity of sound in air to be 340 metres per second, find the time of vibration of the fork. (Matric. 04.)

17. Why, when the handle of a vibrating tuning fork is put in contact with a wooden board, is the amount of sound produced greatly increased? Is the time during which the fork goes on vibrating affected? (Camb. Loc. Jun.)

## CHAPTER VII

### DOPPLER'S PRINCIPLE

§ 32.—The pitch of a musical note is altered if the source or the observer move relatively to one another. Let S (Fig. 26) represent a source of sound, and $O_1$ and $O_2$ observers on opposite sides of it at such a distance that sound takes 1 sec., say, to travel to them. In the figure the sound waves are represented by curved lines, and it will be seen that in the distance OS there are seven waves, and

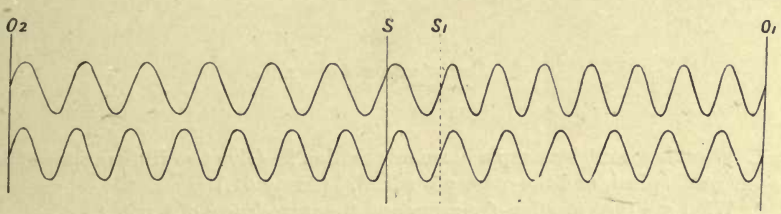



Fig. 26.

as sound covers the distance OS in 1 sec., there will be seven waves per second reaching the observer, *i.e.*, the sound has a frequency of seven. Now let the source move uniformly in the direction $SS_1$, accomplishing the distance in 1 sec. Seven waves are now crowded into the space $S_1O_1$, for the first wave starts from S and reaches $O_1$ in 1 sec. The seventh wave starts 1 sec. later and has only the distance $S_1O_1$ to travel, and therefore gets to $O_1$ in $\frac{S_1O_1}{SO}$ second. The observer at $O_1$ therefore receives seven waves in $\frac{S_1O_1}{SO}$ second, or $7 \times \frac{SO}{S_1O_1}$ waves in 1 sec.

The pitch of the note given out is therefore apparently raised, since more vibrations reach the ear in a given time. Thus a source of sound appears to produce a higher note when moving towards an observer. The reverse takes place with the observer at $O_2$, and the pitch of the sound heard is lowered. It can easily be seen that the same effect takes place when the observer moves relatively to

the source of sound. If he moves towards it he meets more waves in a second than if he remained stationary, and so the sound appears higher; while if he moves away he receives fewer waves and the sound is lowered.

Professor Poynting, at a lecture in Oxford, showed experimentally the raising or lowering of a note by motion of the source. He had a pitch pipe, giving out a certain note, attached to some flexible tubing. By blowing down the tube he was able to sound the pipe. When the pipe was whirled in a circle by means of the tube, at one time it was approaching, at another receding from the audience, and the sound rose and fell in pitch at every revolution.

The pitch of a locomotive whistle can sometimes be heard to change as it passes an observer, being higher when approaching and lower when receding.

*Examination Papers.*

I.

1. What is meant by simple harmonic motion? Give two illustrations of it, —one mechanical, and the other acoustical. (Army 01.)

2. What are the *vibration-frequency* and the *wave-length* of a note?

Describe some form of siren and explain how it could be used to determine the aforesaid characteristics of a note. (Oxf. Prelim. and C. (3) 09.)

3. Explain the cause of beats.

The points of the prongs of a tuning fork A, originally in unison with a fork B of frequency 512, are filed, and the forks produce 5 beats per second when sounded together. What is the pitch of A after filing? (Army 02.)

4. Explain how the pitch of a note can be determined by the monochord. (Army 00.)

5. Find the wave-lengths of the fundamental note and overtones given out by an organ pipe open at one end and closed at the other. (Army 03.)

6. What is the object of sounding boards attached to a stringed instrument, such as a violin? (Army 00.)

II.

1. Give experimental reasons for our belief (1) that sound is due to vibrations, and (2) that air is capable of conveying sound. (Army 02.)

2. Explain the production of echoes, and show how an echo may be used to determine the velocity of sound. (Army 00.)

3. Explain the meaning of the "pitch" and the "intensity" of a musical sound.

How do they respectively depend on the nature of the sound wave which produces the note? (Oxf. Loc. Jun. 04.)

4. Describe the siren, and explain how it is used to determine the frequency of a given note. (Oxf. Loc. Sen. 07.)

5. Explain what is meant by resonance, and give some *mechanical* illustrations of it. (Army 00.)

6. Give illustrations of shadows in sound. Why are sound shadows in general not so well marked as those of light? (Army 03.)

## III.

1. What are the characteristics by which one sound differs from another? How are these characteristics physically related to the motion of the particles by which the sound is transmitted? (Oxf. Prelim. and C. (3) 04.)

2. A man standing by a railway notices that the pitch of the note due to the whistle of an engine diminishes as the engine passes him. Explain this result.

If the frequency of the whistle is 256 vibrations per second and the velocity of the engine is $\frac{1}{20}$ of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him? (Camb. Loc. Sen. 06.)

3. 56 Stimmgabeln lassen sich in eine Reihe so anordeu, dass jede mit der vorhergehenden in der Sekunde 4 Stösse gibt. Die letzte Stimmgabel ist die Oktave der ersten. Berechne die Schwingungszahlen. (Lond. Inter. Sci. 07.)

4. If the handle of a vibrating tuning fork is held against a wooden board, the amount of sound produced is considerably increased. Explain carefully why this happens. Is the time during which the fork goes on vibrating affected, and, if so, why? (Lond. Inter. Sci. 07.)

5. The note corresponding to 256 vibrations per second is sounded on a piano and on a violin. State how the waves from each instrument resemble each other, and how and why they differ.

If the wave-length of this note in hydrogen is 5 metres, find the velocity of sound in that gas. (Oxf. Prelim. and C. (3) 07.)

6. Explain the effect of temperature upon the frequency of the note emitted by an open organ pipe. Does the effect depend upon the material of which the pipe is made or upon the nature of the gas in which the pipe is sounded? Give reasons. (Matric. 08.)

# LIGHT

## CHAPTER I

### INTRODUCTORY.—PHOTOMETRY

§ 1.—The science which deals with the nature and the properties of light and vision is called Optics.

Let us consider what happens when a spiral of, say, platinum wire is heated in a dark room by passing a gradually increasing electric current through it. At first the wire is invisible, but it is radiating out heat-energy which can be detected by any delicate thermoscope, such as a thermopile (*see* § 47, H., § 132, E.C.), placed near it. If the heat is being radiated fast enough it can produce the sensation of warmth on a hand placed near the wire.

Presently, however, as the temperature rises, another phenomenon will appear, for the wire will become visible. The radiations from the wire are now producing the sensation of vision in our eyes as well as the sensation of warmth in our bodies. The wire is, in fact, radiating light as well as heat, and may now be called a *luminous body*. As the temperature rises still higher, the amount of light emitted becomes greater, and its colour changes from dull red to bright red, and finally to white. Other objects in the room, not of themselves luminous, become visible by means of the light received by them from the wire and returned to our eyes. Most bodies are non-luminous, and are only seen because of their power of returning light falling on them from some luminous body such as the sun or a lamp.

Let us assume, then, that a luminous body is a source of energy which is being radiated out in all directions, and that the phenomena of light and heat are manifestations of this energy. The question immediately arises, How is this energy propagated from the luminous body to the observer? Two theories have been proposed: first, the emission theory which conceives a luminous body as continually shooting forth extremely minute material particles in all directions with enormous velocity; second, the wave theory which supposes light to be propagated in very tiny wavelets travelling with very

great velocity through the *æther*, "a hypothetical medium permeating all space." Of these two theories the first will account for many of the general phenomena of light, but fails utterly to explain certain facts. For example, light on this theory should travel more rapidly in glass than in air—a conclusion which is directly opposed to experimental results (*see* § 92).

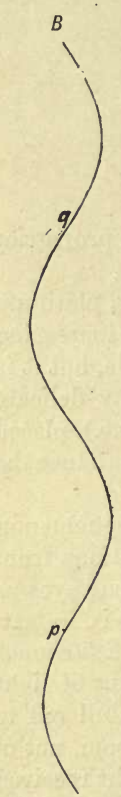


Fig. 1.

The wave theory has, however, provided a satisfactory explanation of all facts so far observed, and is now universally accepted.

We shall, therefore, assume that light travels through the æther in very tiny wavelets produced (somewhat after the manner of ripples in water) by vibrations in the æther at right angles to the direction in which the light is travelling.

In order to get some idea of the way in which transverse waves are propagated, let us consider a long flexible cord AB, one end B of which is fixed while the other end A is free to move.* If the end A is quickly displaced from A to $A_1$ and back through A to $A_2$ and then continuously from $A_2$ to $A_1$ and back again, the displacements given to A will travel along the cord as a wave. Each point of the cord simply moves backwards and forwards just as A does, but a disturbance travels along the cord from A to B. Take any point $p$ of the cord and let $q$ be the next point to it which has the same displacement and the same direction of motion as $p$. Then $p$ and $q$ are said to be in the same phase of the vibration, and the distance from $p$ to $q$ is called a wave-length. The distance $AA_1$, *i.e.*, the greatest displacement of any particle from its undisturbed position, is called the amplitude of the vibration (*see* pp. 1—5).

In order to form an idea of the propagation of waves in a continuous medium, we need only imagine an infinite number of cords, touching one another, and all vibrating in the same manner. Thus Fig. 2 represents the propagation of plane waves in a medium, and Fig. 3 shows a series of spherical waves diverging from a luminous point.

* A piece of rubber tube is really better for this experiment than a cord.

A surface like *pqr* in either of these figures, every point of which is in the same phase of vibration, is called a wave-front. It should be clearly understood that the æther waves which produce light are extremely minute (the wave-length is measured in ten-thousandths of a millimetre) and that they travel with enormous velocity (about three hundred million metres a second). Moreover, the particles need not, as in the illustrations, vibrate in straight lines, but may describe any curves in planes at right angles to the direction in which the light is travelling.

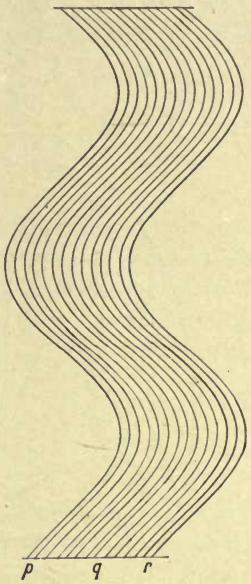


Fig. 2.

§ 2. *The Velocity of Light.*—When the cord of § 1 was shaken at one end so that waves travelled along it, a wave starting from one's hand took a definite, though small, time to travel to the other end of the cord. The rate of propagation of the wave could have been found by measuring the length of the cord and the time taken for a wave to traverse it from end to end. When the velocity of light is spoken of it is this velocity of propagation that is meant, and no reference is made to the time which any one æther particle may take to make a complete vibration.

If a very distant source of light is suddenly kindled and afterwards extinguished, there would be an appreciable interval between its first kindling and its appearance to our eyes, and the light would appear visible for a short time after it was really extinguished. The first suggestion that light has a definite velocity was made by Römer in 1656, as the result of his observations on a satellite of Jupiter.

Fig. 4 gives a diagrammatic sketch of the sun, the earth and the planet Jupiter. When the earth and Jupiter were in the position of Fig. 4A Römer observed that Jupiter's satellite made a complete revolution in 48 hours 28 mins. 35 secs. It was therefore easy to calculate the times at which the satellite should disappear behind Jupiter.

When, however, the earth, sun and Jupiter were in the relative positions of Fig. 4B, the observed time of eclipse was always 16·5

mins. later than the calculated time. This was not due to errors of observation, since the observed times coincided with those calculated when the planets had once again come into the positions of Fig. 4A.

Suppose the light from Jupiter takes a time $T$ seconds to reach the earth when their relative positions are those of Fig. 4A; then when the planets are in the positions of Fig. 4B, the time will be $T + 990$ secs. If J is the distance of Jupiter from the sun, and

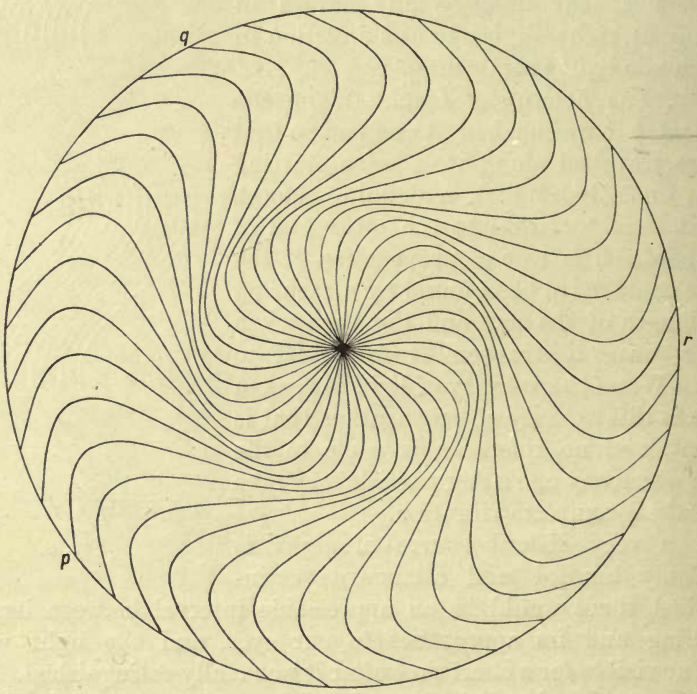



Fig. 3.

E is the distance of the earth, and $V$ is the velocity of light, then since the earth is 2E further from Jupiter in the position of Fig. 4B than in the position of Fig. 4A light will take $\frac{2E}{V}$ secs. longer to reach the earth from Jupiter. Therefore $\frac{2E}{V} = 990$ secs.

Now 2E = twice the distance of the earth from the sun,
= 296 millions of kilometres, or 182,000,000 miles.

Therefore the velocity of light

$= \frac{296{,}000{,}000}{990}$ kilometres per sec., or $\frac{182{,}000{,}000}{990}$ miles per sec.,

= 299,000 kilometres, or 186,000 miles per second.

There are other methods by which this velocity may be determined, and these will be considered later (*see* §§ 90 to 92).

§ 3. *Interference.*—When several different disturbances affect the

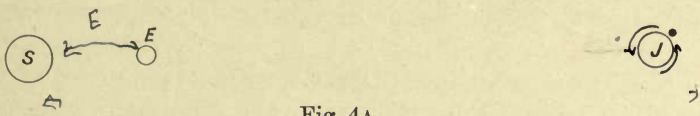



Fig. 4A.

same portion of a medium simultaneously, the resulting vibration of the medium is obtained by adding together the displacements due to the separate vibrations. For example, if the vibrations repre-

Fig. 4B.

sented by the continuous curves in Fig. 5 were superimposed on the same medium in the same plane, the resultant disturbance would be that shown by the dotted curve. In this case the separate displace-

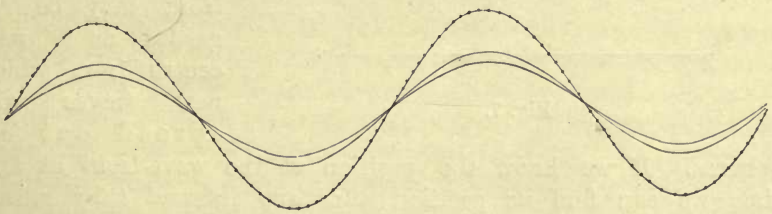


Fig. 5.

ments reinforce one another, and the amplitude of the resulting vibration is greater than that of either of the component vibrations.

Fig. 6 shows the resultant of the same two vibrations in the same plane when one is half a wave-length (or any odd number of half wave-lengths) behind the other. The separate displacements are

now opposed to one another, and the effect is that the amplitude of the resultant vibration is comparatively small.

If the amplitudes of the component vibrations in Fig. 6 had been exactly equal, then there would evidently have been no resultant vibration, and the two superimposed wave systems would have destroyed one another. From such considerations as these, we

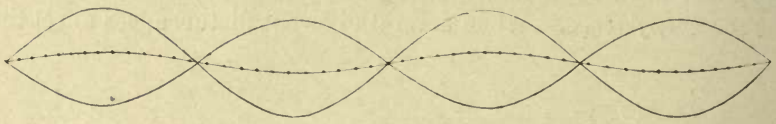


Fig. 6.

arrive at the idea of the superposition of two lights destroying, or partially destroying, one another and producing darkness; the conditions for this are that the two systems have the same wave-length and amplitude, but that one is retarded upon the other by an odd number of half wave-lengths.

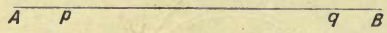



Fig. 7.

This phenomenon is called *Interference*.

§ 4. *Huyghens' Principle.* — Each point of any wave-front may be regarded as a new centre from which new waves will spread out in spheres. If we know the position of the wave-front at any time we can find its position after any interval $t$ by means of this principle. For example, let AB (Fig. 7) represent the position of the front of a plane wave at any time. Calculate the distance through which the disturbance would travel in the interval $t$; and with centres at various points $p$, $q$, etc., on AB and radius equal to this distance describe spheres. The surfaces of these spheres are where the disturbances originating at $p$, $q$, etc., would have spread to in the time $t$, and the plane A'B' which

touches them all represents the new position of the wave-front. Then the only portions of the little waves originating from *p*, *q*, etc., which are effective in forming the new wave-front can be shown to be those which touch A′B′, the other parts interfering with and neutralising one another.

§ 5. *Terminology.*—A substance such as air, glass, water, crystal or gelatine through which light can pass is called a *Medium.*

If the medium has the same properties at all points and in all directions it is called *Homogeneous.*

A body through which objects can be seen distinctly is said to be *Transparent.*

A body through which light cannot pass is said to be *Opaque.*

In general when light falls on any body, one portion of the light is *Reflected*, one portion is *Transmitted*, and the rest is *Absorbed.*

For example, when light falls on a bright polished surface, such as a smooth burnished piece of silver, most of it is reflected regularly according to the laws considered later (§ 13). When light falls on a piece of white paper some is reflected regularly, and some is reflected irregularly or diffused owing to minute roughnesses of the surface; it is by this diffused light that the paper becomes visible.

When light falls on the surface of a plate of glass, very little is absorbed, some is reflected, but most of the light is transmitted through the glass; while when light falls on a dull black surface such as that of lamp-black it is almost wholly absorbed.

There are three fundamental laws or principles on which the elementary science of optics depends. These are: (1) the rectilinear propagation of light; (2) the law according to which light is regularly reflected from a smooth reflecting surface; and (3) the law in accordance with which light travels when it passes from one homogeneous medium to another, or the law of refraction.

§ 6. *The Rectilinear Propagation of Light.*—If light is admitted into a dark room from the sun or a lamp through a small hole, and if a piece of paper is held in the room so that the lamp, hole and paper are on a straight line, the paper will be illuminated, and a spot of light will be seen on it; but if the paper is not on the straight line from the lamp through the hole no spot of light will be seen on the paper. If the air of the room be dusty, or if there be smoke in it, the path of the beam can be seen by the light reflected from the dust particles. It will be seen that this beam is quite straight across the room and that the straight line joining

the lamp to the hole can be produced down the centre of the beam.

Suppose that S (Fig. 8) is the hole in a screen through which light is admitted and that O is a point opposite to it. When the light arrives at the screen all the waves will be stopped except that little piece of each which comes to S. That bit will become a new origin of disturbances sending out fainter spherical waves all round. Consider any wave-front QPR of the new waves and let P be the point in it nearest to O. Then it can be proved on the wave theory that, owing to the smallness of the wave-lengths in comparison with the other distances involved, only a small part of the wave-front around P is effective in illuminating O, the effects of the remaining portions neutralising one another by interference. Hence, a small opaque body at P would cut off all the light from O, and this is what we mean when we speak of the propagation of light in straight lines, or when we talk of rays of light. A ray of light to any point in a surface may be understood to mean the straight line along which the piece of the wave-front advances which is effective in illuminating that point. The small group of rays from the source of light to any small area of an illuminated surface is called a *pencil* of light.

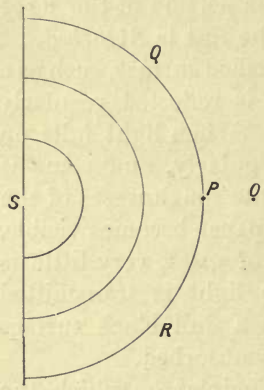


Fig. 8.

§ 7. *Images produced by Small Apertures.*—Make a small hole in a sheet of metal or cardboard; place a lighted candle behind the hole in a dark room and a screen in front of it. Shade the candle so that light may not be reflected from it to the screen by the walls of the room. A faint picture or image of the candle will then be seen upside down on the screen. This is a direct consequence of the rectilinear propagation of light. The very thin pencil from the point *A* of the flame (Fig. 9) which gets through the hole forms a very small patch of light on the screen at *a*. Similarly there is a tiny patch of light on the screen corresponding to each point of the candle, and the assemblage of these patches on the screen forms an image of the flame.

§ 8. *Shadows.*—Suppose that instead of having only one hole in

the card several were made. We should then get several images of the candle side by side and placed in groups similar to the grouping of the holes. Thus, if we have four holes at the corners of a square,

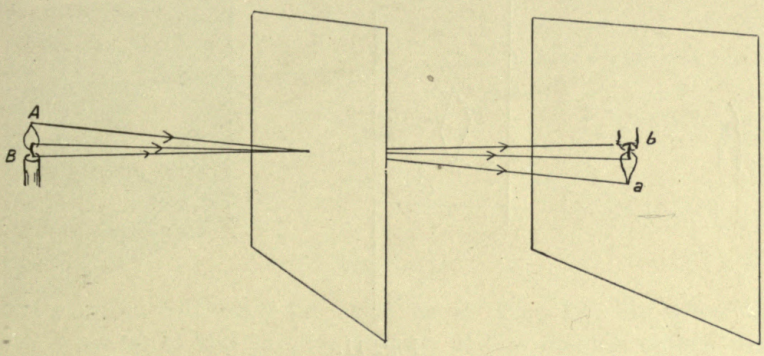



Fig. 9.

on the screen we shall have four images at the corners of a square as in Fig. 10.

By increasing the number of holes we should increase the number of images, and eventually some would overlap. We should thus get a

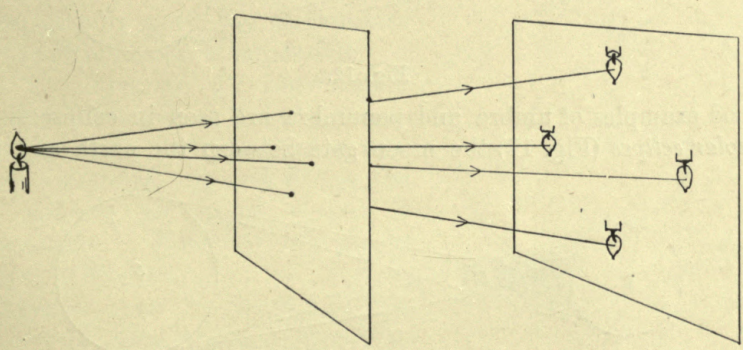


Fig. 10.

blur of images whose general outline would be the same as the general outline of the group of holes. If eventually we cut out a square opening (Fig. 11) in the first screen we should have a square patch of light on the second screen, but its edges would not be sharp, since light from the tip of the flame could reach points low

down on the second screen on which no light from the bottom of the flame could fall. The dark outer parts of the second screen are in *shadow* (or *umbra*), the dimly lighted edges of the central

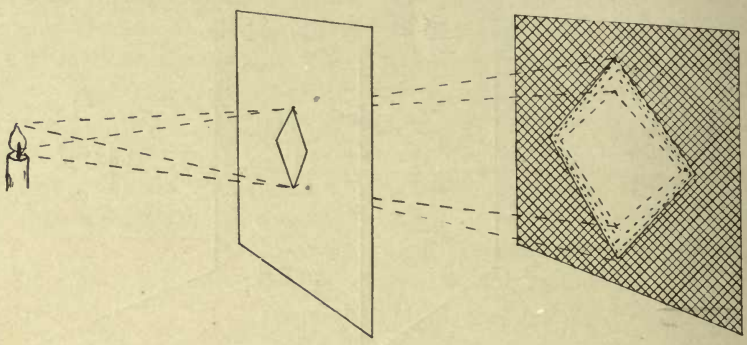


Fig. 11.

square of light are the half-shadow or *penumbra*. Obviously the smaller the source of light the narrower will be the penumbra.

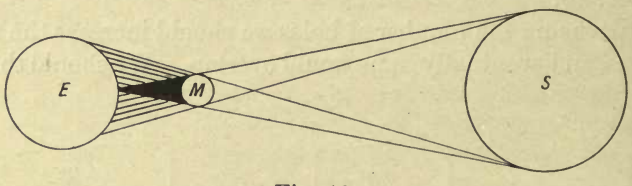



Fig. 12.

Good examples of umbra and penumbra are seen in eclipses. In a *solar eclipse* (Fig. 12) the moon gets between the earth and the

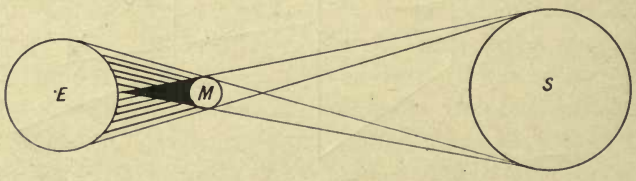



Fig. 13.

sun. The sun is large compared with the moon, and consequently the moon's shadow cannot cover more than a very small portion of the earth at one time; its penumbra can, however, extend over a fairly large area. Those who are in the penumbra experience partial darkness and see only part of the sun's disc. The

relative sizes of the sun, moon and earth are such that a very slight change in their relative distances makes it sometimes possible and sometimes impossible for any portion of the moon's umbra to touch the earth. If the earth is as far from the sun as possible, then the

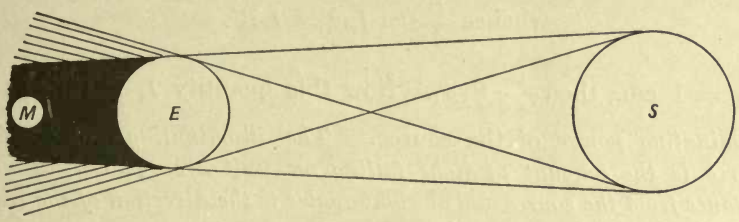



Fig. 14.

moon's umbra will touch the earth as in Fig. 12; but when the earth is nearer to the sun, no portion of the moon's umbra may touch the earth. Persons on certain portions of the earth's surface will then see the edges of the sun only, since the middle parts are hidden by the moon. When this occurs the eclipse is said to be *annular* (Fig. 13).

In a *lunar eclipse* the earth is between the sun and the moon (Fig. 14). The earth's shadow may either cover the entire disc of the moon or only part, but an annular eclipse of the moon is impossible.

§ 9. *Intensity of Illumination. Candle Power.*—Let S be a source of light such that its rays spread in all directions. Round S describe two spheres of radius $r$ and R respectively. Those are shown in section in Fig. 15.

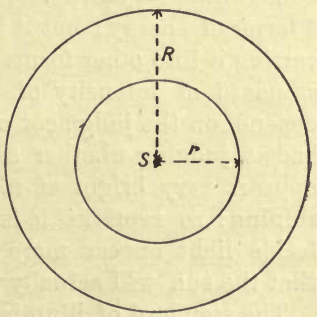



Fig. 15.

Suppose that S represents the amount of light starting from the source. Then the amount falling on unit area of the inner sphere will be S ÷ the area of the sphere = S ÷ $4\pi r^2$. Now the amount of light falling on unit area is called the *Intensity of Illumination*. If $I_r$ is the average intensity on the sphere of radius $r$ we have

$$I = \frac{S}{4\pi r^2}.$$

Similarly if $I_R$ is the intensity of illumination on the surface of the larger sphere, then

$$I_R = \frac{S}{4\pi R^2},$$

$$\text{whence } \frac{S}{4\pi} = I_r r^2 = I\ R^2.$$

If $r = 1$ cm., then $\frac{S}{4\pi} = I_1$. Now this quantity $I_1$ is called the *illuminating power* of the source. *The illuminating power of a source is the amount of light falling on unit area placed at unit distance from the source and at right angles to the direction of the rays.*

We may write our equations—

$$I_1 = I_r r^2 = I_R R^2,$$

$$\text{or } I_r = \frac{I_1}{r^2} \text{ and } I_R = \frac{I_1}{R^2}.$$

*Or the intensity of illumination at any distance from a source of light is equal to the illuminating power of the source divided by the square of the distance from the source.*

So far we have spoken of amount of light as though this were a measurable quantity. Since light is a wave motion, it is also a form of energy; but it is not practical to measure light by converting it into other forms of energy. We must, therefore, have a standard of intensity of illumination. This standard must not depend on the judgment of the eye, for the eye is extraordinarily bad at judging whether a source be bright or not. A lamp which appears very bright at night seems very feeble when the sun is shining; so remarkable is the effect of a bright light in making a feeble light appear more feeble, that popular superstition asserts that the sun will actually put a fire out.

The standard of illuminating power in England is supposed to be a particular kind of candle; but it is in practice a lamp burning pentane, invented by Dr. A. Vernon Harcourt, which gives as much light as five standard candles. A lamp which is found to give ten times as much light as a standard pentane lamp would be said to be of 50 candle-power.

To compare the illuminating power of two sources, use is made of the mathematical law, proved above, that the intensity of illumination at a distance $r$ from a source whose illuminating power is $I$

is $\frac{I}{r^2}$. If we have two sources whose illuminating powers are $I$ and $I'$, and find at what distances they give the same intensity of illumination, we can readily compare $I$ and $I'$; for if a screen placed $r$ cms. from the first lamp is illuminated as brightly as when it is $r'$ cms. from the second lamp, then

$$\frac{I}{r^2} = \frac{I'}{r'^2}, \text{ whence } \frac{I}{I'} = \frac{r^2}{r'^2}.$$

As an example we may suppose that a certain electric lamp produces the same intensity of illumination when it is 50 cms. from a screen as does a (standard) pentane lamp 100 cms. from the screen. Then if $I$ is the candle-power of the electric lamp,

$$\frac{I}{5} = \frac{(50)^2}{(100)^2} = \frac{1}{4},$$

$\therefore I =$ the candle-power of the lamp $= \frac{5}{4} = 1\frac{1}{4}$ standard candles.

The next question to be considered is, How can we judge whether the intensity of illumination produced by a given lamp (A) at a certain distance is equal to, greater than, or less than that produced by another lamp (B)? It would be possible to put lamp A at a certain distance from a screen, and, by moving some opaque body, to cut off the light from A and allow that from B to fall on the screen. Our judgment in this case would rely on memory, and the eye has an extraordinarily bad memory for intensity of illumination. It is practically necessary that one should see a screen, part of which is illuminated by A and part by B. This is secured in *Rumford's Rod Photometer*.

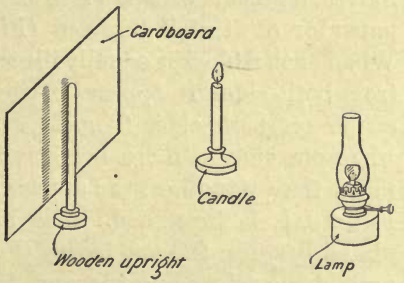


Fig. 16.

§ 10.—Suppose that we wish to compare a lamp with a candle. A rod (Fig. 16) is placed in front of a white screen. The lamp and candle are so placed that the shadows cast by them lie side by side on the screen.

Looking down from above, we then have the arrangement shown in Fig. 17. If this is examined it will be seen that the "shadow *a* cast by B" is really that part of the screen which receives no light

from B, and is therefore illuminated only by A. Similarly the "shadow cast by A" is the part of the screen illuminated only by B. If the lamps are so arranged that the shadows are equally deep, then

$$\frac{I_A}{(\mathrm{A}a)^2} = \frac{I_B}{(\mathrm{B}b)^2}.$$

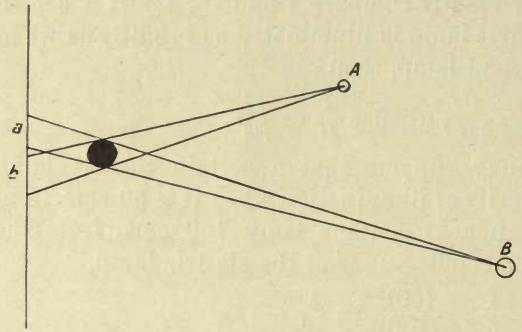



Fig. 17.

Notice that the distances measured are from A to B's shadow and from B to A's shadow, and not from A to A's shadow.

§ 11. — Another form of photometer is made by cutting a block of paraffin wax through the middle, and sealing the two halves together with a piece of brown paper or of tinfoil between (Fig. 18). When such a block is equally illuminated from both sides it appears of the same colour on both sides (Fig. 18, *a*). But when one side is more brightly illuminated than the other, the appearance of Fig. 18, *b*, is presented. The block is placed between the two lamps (Fig. 19), and is moved backwards and forwards until both halves appear equally bright when viewed from the side.

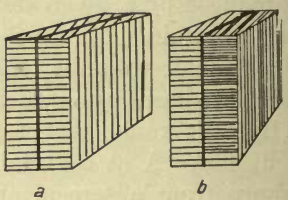



Fig. 18.

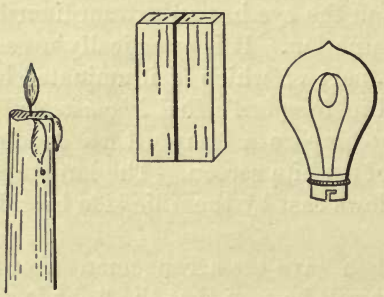


Fig. 19.

The distances of the lamps from the centre of the block are then measured, and the relative intensities calculated by the inverse square law as before.

§ 12.—Instead of the block, *Bunsen's Grease Spot* may be used. To make this, a sheet of thin paper is well stretched on

a frame ; a small spot of grease is then dropped on to it, and partly soaked out again by the use of warm blotting-paper. One's aim should be to leave the spot more transparent than the rest of the paper, but without a shiny surface. When the paper is held between the observer and a source of light, the grease spot appears brighter than the surrounding paper ; but when the paper is viewed by reflected light the spot appears darker than the rest of the paper. If the paper is moved about between two lamps, just as the paraffin block was moved, until the grease spot is indistinguishable from the rest of the paper, then the effect of the light coming through (from A) must be the same as the effect of the reflected light (from B). Hence the screen is equally illuminated by both sources, and the candle-powers of these can be compared.

*Examples.*

1. Show how the velocity of light has been determined by observations on the satellites of Jupiter. (Army 04.)

2. Explain the formation of the picture of external objects seen on a screen in a dark room near a small hole in the shutter.

What is the effect of (i.) changing the size of the hole, (ii.) changing the distance of the screen from the hole ? (Camb. Spec. 07.)

3. On what evidence does the statement that light is propagated in straight lines depend ? Describe the phenomena observed in a total eclipse of the moon. (Camb. Loc. Jun. 07.)

4. What is meant by the statement that a certain lamp is of 16 candle-power ? How would you test the accuracy of the statement ?

It is found that a 16 candle-power lamp produces the same intensity of illumination on a sheet of paper 8 ft. from it as an arc lamp which is 60 ft. from the paper. Find the illuminating power of the arc lamp. (Matric. 04.)

5. Explain the principle of the Shadow Photometer, and state how it could be used to find how the intensity of illumination depends on the distance from the source of light. (Army 02.)

6. In a darkened room, a lamp and a standard candle, distant 10 and 15 decimetres respectively from a vertical stick, cause shadows of equal intensity to fall side by side upon a screen 20 cms. behind the stick. Find the candle-power of the lamp. (Army 02.)

7. What do you understand by (i.) intensity of illumination, (ii.) illuminating power ? Explain how in ordinary photometers the second of these is measured by means of the first. (Camb. Loc. Sen. 07.)

8. A 10 candle lamp is placed 1 metre from a surface. At what distances must gas flames of 14 and 16 candle-power respectively be placed so as to produce an equal illumination of the surface ? Mention any one method that is commonly used to enable equality to be judged. (Lond. Inter. Sci. 03.)

9. Explain what is meant by the *candle-power* of a lamp, and describe a simple method by which it can be approximately determined.

Two lamps are placed on opposite sides of a screen, and their distances from the screen so adjusted that the two faces of it are equally illuminated. A semi-transparent sheet is then placed between one of the lamps and the screen, and it is found that the other lamp must be moved to twice its original distance from the screen in order that the two faces may be equally illuminated again. What fraction of the light falling upon it is cut off by the sheet? (Matric. 08.)

10. How would you compare the illuminating powers of two sources of light for some particular colour? (Camb. Loc. Sen.)

## CHAPTER II

### REFLECTION

§ 13. *Laws of Reflection.*—In § 5 it was pointed out that light falling on a surface may be scattered or reflected. In future when we speak of reflection we shall mean that kind which is observed in the case of a polished sheet of metal or of a mirror.

In considering optical phenomena, a great use is made of what is termed the *angle of incidence*—that is, the angle between the incident ray and the normal to the bounding surface of two media.

Thus, suppose that QP (Fig. 20) represents a ray meeting the

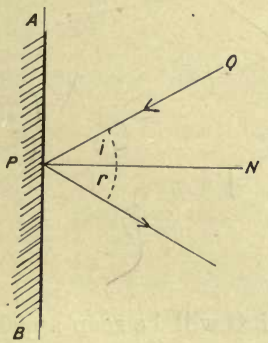



Fig. 20.

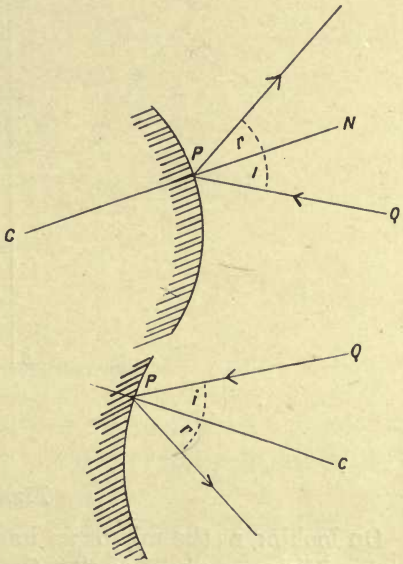



Fig. 21.

bounding surface AB between two media at the point P. Then, if PN is drawn at right angles (or normal) to AB, the angle QPN is the angle of incidence. If the surface is spherical, as in Fig. 21, the normal will be a radius (since the radius and tangent are at right angles), and the angle of incidence is QPN or QPC.

When light undergoes reflection we have also to consider the *angle of reflection*—that is, the angle between the normal and the reflected ray (marked *r* in the figures).

*The law of reflection is that the angle of incidence and the angle of reflection are equal and lie in one plane.*

§ 14.—This may readily be proved by the following simple experiment:—

Set up a strip of mirror vertically on a drawing-board, and in front of it, at about 20 cms. from the mirror, stick in a pin. The mirror and pin are shown by AB and O in Fig. 22 as though looked down on from above.

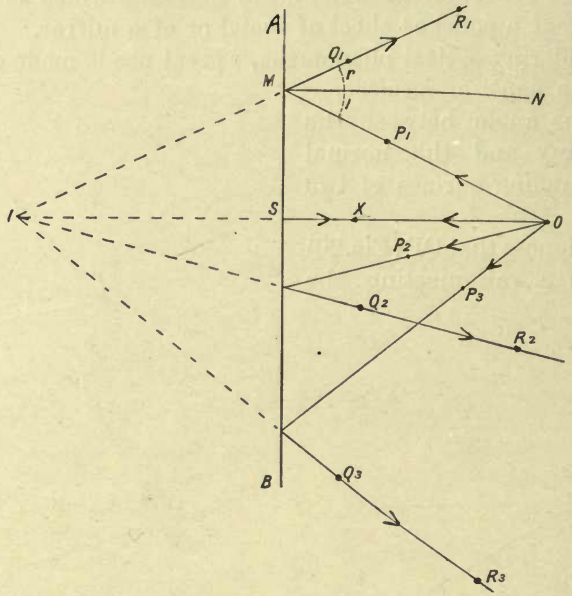



Fig. 22.

On looking at the mirror an image of O will be seen apparently at I. Stick in a pin X so that O, X and I appear in a straight line.

Now stick in a pin in some such place as $P_1$ and stick in pins $Q_1$ and $R_1$ in line with the images of O and $P_1$ seen in the mirror (in line with the *images*, not with the pins themselves).

Stick in a pin at $P_2$ and place $Q_2$ and $R_2$ in line with the images of O and $P_2$. Other pins $P_3$ $Q_3$ $R_3$, $P_4$ $Q_4$ $R_4$, etc., should be stuck in similarly; some of these are shown in the figure.

After removing the mirror draw in the paths of the rays; a ray starting from O and passing through $P_1$ goes, after reflection, along $Q_1R_1$, etc.; the ray OXS comes back along SXO.

If $R_1Q_1$, $R_2Q_2$, etc., are produced, it will be found that they meet at a point I on OX produced, and that I is as far behind the mirror as O is in front.

All the rays after reflection at a plane mirror come as though from a point as far behind the mirror as their point of origin is in front. This point is called the *image*. I is the image of O.

We may say, therefore, that the image is as far behind a plane mirror as the object is in front.

Now consider any one of the rays $OP_1Q_1R_1$ meeting the reflecting surface in M. Draw the normal MN at M.

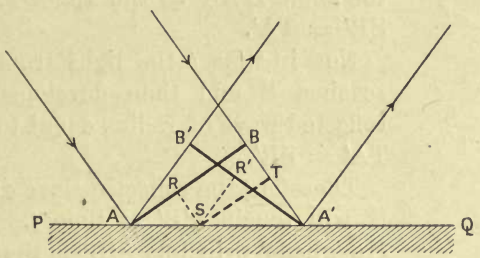



Fig. 23.

In the triangles IMS, OMS

IS = SO,
SM is common,
angles at S are right angles,
therefore the angle OMS = the angle IMS
= the opposite angle $AMQ_1$,
therefore the angle OMN = the angle $R_1MN$,
or the angle of incidence = the angle of reflection.

§ 15. *Theoretical.*—If we attempt to represent this law on the basis of the wave theory, we find that theory and fact are in entire agreement.

Let AB (Fig. 23) be the front of a plane wave arriving at a reflecting surface PQ, and let BA′ be a ray perpendicular to AB.

Let $t$ be the time taken by the light from B to reach A′. Then in the time $t$ the reflected wave at A will have developed into a sphere of radius BA′.

With centre A and radius = BA′ describe a sphere and let A′B′ represent a tangent plane to this sphere through A′.

Then shall A′B′ represent the front of the reflected wave at the instant when the light from B reaches A′.

For in the triangles ABA′ and AB′A′, A′B = AB′, being the distances which light travels in time $t$. AA′ is common, and the angles at B and B′ are right angles, therefore the angles BA′A and B′AA′ are equal.

Now consider a ray RS perpendicular to AB from any point R in AB.

Draw ST perpendicular to BA′ and SR′ perpendicular to B′A′.

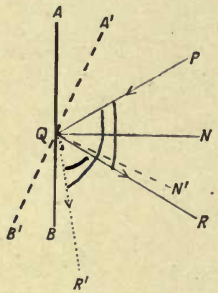


Fig. 24.

Then the triangles STA′ and A′R′S are equal in all respects, for the angle R′SA′ = the angle B′AA′ = the angle TA′S, therefore SR′ = TA′.

Now in time $t$ the light from R will have reached S and then developed a spherical reflected wave of radius equal to BA′−RS = TA′ = SR′.

Therefore the reflected wave diverging from S will touch A′B′ at time $t$. Therefore by Huyghens' principle A′B′ is the wave-front at time $t$. Now the angle BAA′ = the angle between any incident ray and the normal to the surface at the point of incidence, *i.e.*, it is equal to the angle of incidence.

Similarly the angle B′A′A is equal to the angle of reflection.

Therefore the angle of incidence is equal to the angle of reflection.

§ 16. *Effect of Turning a Mirror on the Reflected Ray.*—Let AB (Fig. 24) be a mirror and PQ any ray incident at Q. If NQ is the normal at Q, then QR will be the reflected ray where angle PQN = angle NQR.

Now turn the mirror to such a position as A′B′. The normal at Q will now be QN′, and the angle NQN′ will be equal to the angle AQA′.

The reflected ray will now be QR′ where R′QN′ = PQN′.

Now the reflected ray is turned through the angle R′QR by the rotation of the mirror.

But angle R′QR = angle R′QP − angle RQP
= twice angle N′QP − twice angle NQP
= 2(N′QP − NQP)
= twice angle NQN′
= twice angle AQA′.

Or the reflected ray is turned through twice the angle through which the mirror is turned.

§ 17. *The Box Sextant* (Fig. 25) depends on this principle. The mirror $M_1$ is fixed and has a vertical line drawn down it. Another mirror $M_2$ is fastened to a movable arm $PM_2$. At O is a peep-hole. An eye looking through O, will be able to see light from a distant object A on the line $OM_1$ produced. By looking down a little the same eye can see light reflected from the mirror $M_1$. This mirror is set at such an angle that the ray which is reflected along $M_1O$ is incident along the line $M_2M_1$. Suppose that the ray comes to $M_2$ from a distant

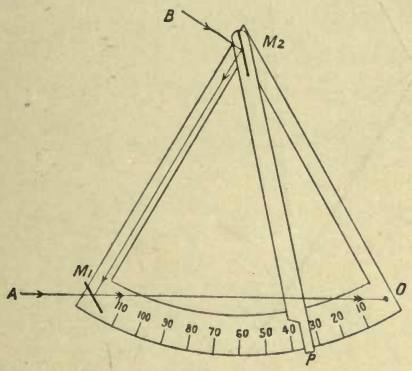



Fig. 25.

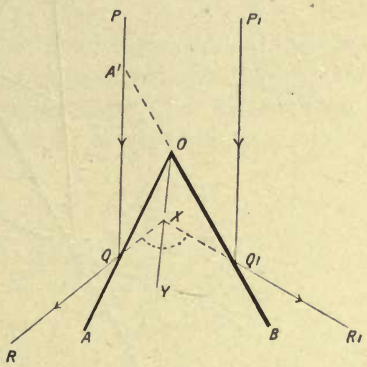



Fig. 26.

object B. If $BM_2$ is parallel to $AM_1$, the pointer P is at the zero mark; but if $BM_2$ makes an angle $\theta$ with $AM_1$ then the mirror $M_2$ must be turned through $\frac{1}{2}\theta$ in order that the reflected ray from $M_2$ may fall on the centre line of $M_1$. The divisions on the scale over which P moves are therefore of the size of half-degrees, but are marked as whole degrees, and thus the angle between $BM_2$ and $AM_1$ can be read off directly.

§ 18. *Parallel Rays falling on Two Mirrors at an Angle to one another.*—Let OA, OB (Fig. 26), be two mirrors inclined to one another at an angle AOB, and let PQ be a ray falling on OA at Q, and reflected along QR. Let $P_1Q_1$ be a ray parallel to PQ, but falling on OB and reflected along $Q_1R_1$.

If the mirror AO were rotated about O until it lay along the line OA', the ray QR would be rotated until it was parallel to $Q_1R_1$.

But we know that the angle through which QR is turned is twice that through which AO is turned; that is $360° - RXR_1 = 2(180° - AOB)$; therefore angle $RXR_1$ = twice angle AOB.

Or the angle between the reflected rays = twice the angle between the mirrors.

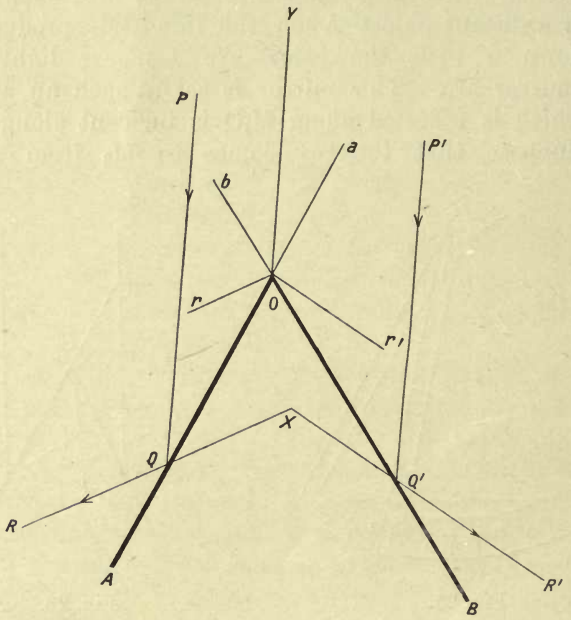



Fig. 27.

A more direct proof of this is as follows (Fig. 27):—At O draw OY, O*r* and O*r'* parallel to QP, QR and Q'R' respectively.

Produce AO and BO to *a* and *b*.

Angle RQA = angle *r*OA,
and angle PQO = angle YO*a*,
∴ angle *r*OA = angle YO*a*;
similarly angle *r'*OB = angle YO*b*,
therefore angle *r*OA + angle *r'*OB = angle *a*O*b*,
= angle AOB,
and angle *r*O*r'* = twice angle AOB,
or angle RXR' = twice angle AOB.

§ 19. *Parallax.*—Hang a pear-shaped weight by a string, so that it is about 2 cms. from the table and point downwards. Stick

a pin into the table immediately under the weight. From whatever point these two objects are looked at the pin appears under the tip of the weight. Move the pin somewhat nearer to yourself; it will now appear under the weight when the pin, your eye and the weight are in a straight line; but if you move your head to the right the weight will appear to the right of the pin. When your head moves to the left the weight appears to the left of the pin. This illustrates the principle of parallax, which may be stated thus: if two objects are in line with the eye, but not at the same distance from it, the further object appears to move relatively to the nearer object in the same direction as the observer's head when this is moved from side to side.

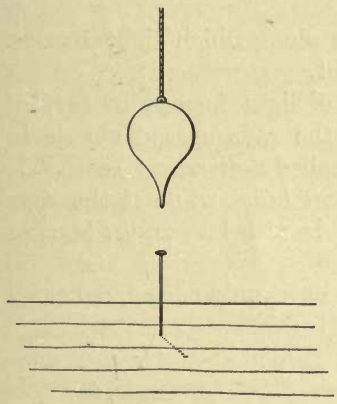


Fig. 28.

Take the drawing board and mirror used in § 14, stick a pin in at O (Fig. 22), and set up the mirror along AB as before. Put in another pin behind the mirror

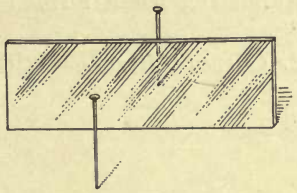


Fig. 29.

near to I. By looking over the top of the mirror the head of the second pin can be seen at the same time as the lower half of the image of the first pin as in Fig. 29. By using the principle of parallax a position can be found for the second pin, such that its head appears continuous with the image of the first pin from whatever point they may be viewed. This position will be that of the image of the first pin, and will be found to be the same as the point I found in § 14.

§ 20. *Images.*—The image which we have just found is merely a point from which rays *appear* to come after reflection. Such an image is called a *virtual* image. In some optical instruments which we shall consider later the rays actually pass through the image. In the photographic camera the rays actually go to the image on the sensitive plate; in the optical lantern (or magic

lantern) the rays go to an image on the screen. When the light actually goes to the image the image is said to be *real.*

These are the only kinds of images.

§ 21. *Rays and Beams of Light.*—In § 14 an experiment was described in which rays of light were traced; and in § 6 it was

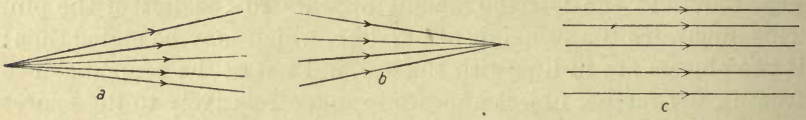



Fig. 30.

pointed out that a ray of light is a line along which light travels. A bundle of rays is called a beam of light.

In Fig. 30 are shown some *beams* of light formed by rays of which some are represented. When the rays spread out as in Fig. 30*a* the beam which they form is called a *divergent* beam. If the rays converge they form a *convergent beam*, while if the rays are parallel the beam formed by them is called a *parallel beam* of light.

§ 22. *Curved Mirrors.*—Hitherto we have only considered cases

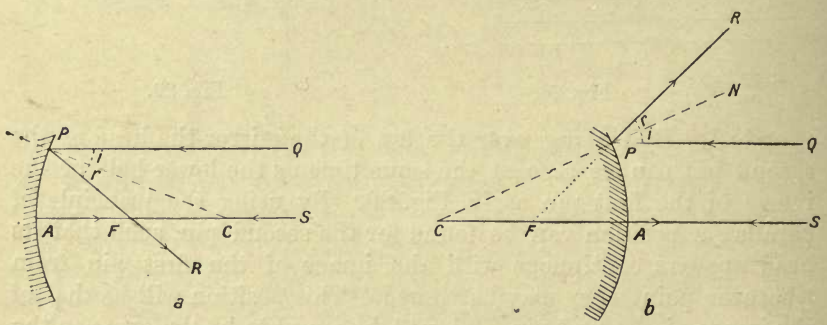



Fig. 31.

of reflection at plane surfaces; but if the reflecting surface is curved the law is the same, the angles of incidence and reflection are equal and lie in the same plane. But since the normals at different points on the surface are not parallel the way in which images are formed is not quite so simple as in the case of a plane mirror. We shall only consider here those curved mirrors the

surfaces of which form parts of spheres; any section of such a mirror is part of a circle.

There are two kinds of curved mirror; in one (Fig. 31*a*) the centre of curvature is in front of the mirror; such a mirror is called a *concave mirror*. In the other kind the centre of curvature is behind the mirror; such a mirror is said to be *convex* (Fig. 31*b*).

§ 23.—Before dealing further with these mirrors it is necessary to adopt a *Rule of Signs*. All lengths will be measured from the surface of the mirror; those which are measured in front of the mirror are positive, distances behind the mirror are negative.

§ 24.—In Figs. 31*a* and 31*b* are represented sections of two spherical mirrors. In each case C is the centre of curvature, and CA is the radius of curvature. In Fig. 31*a* CA $= +r$, in Fig. 31*b* AC $= -r$, in accordance with our rule of signs.

If SA is a ray of light meeting the mirror in A, which either passes through C (Fig. 31*a*) or would pass through C if produced (Fig. 31*b*), then SA strikes the mirror normally and is reflected back along the same path AS.

Suppose QP is another ray parallel to SA striking the mirror at P. Draw the normal CP. Then the reflected ray PR must make the same angle with CP as QP does. We can thus construct the direction of PR geometrically.

In Fig. 31*a* we see that PR cuts CA in F, and in Fig. 31*b* PR proceeds as though from a point F in CA.

Since QP is parallel to CA

$$\therefore \angle i = \angle \mathrm{PCF} \text{ in both figures,}$$

and in Fig. 31*b* $\angle r =$ vertically opposite angle CPF,

$\therefore$ in both figures

$$\angle i = \angle \mathrm{PCF} = \angle r = \angle \mathrm{CPF},$$

$\therefore$ in the triangle CFP

$$\text{side FC} = \text{side FP}.$$

Now so long as P is near to A the lengths FP and FA will be very nearly equal,

$\therefore$ FC is very nearly equal to FA,

or F is very nearly the middle point of CA.

Whence we may conclude that all rays parallel to CA proceed,

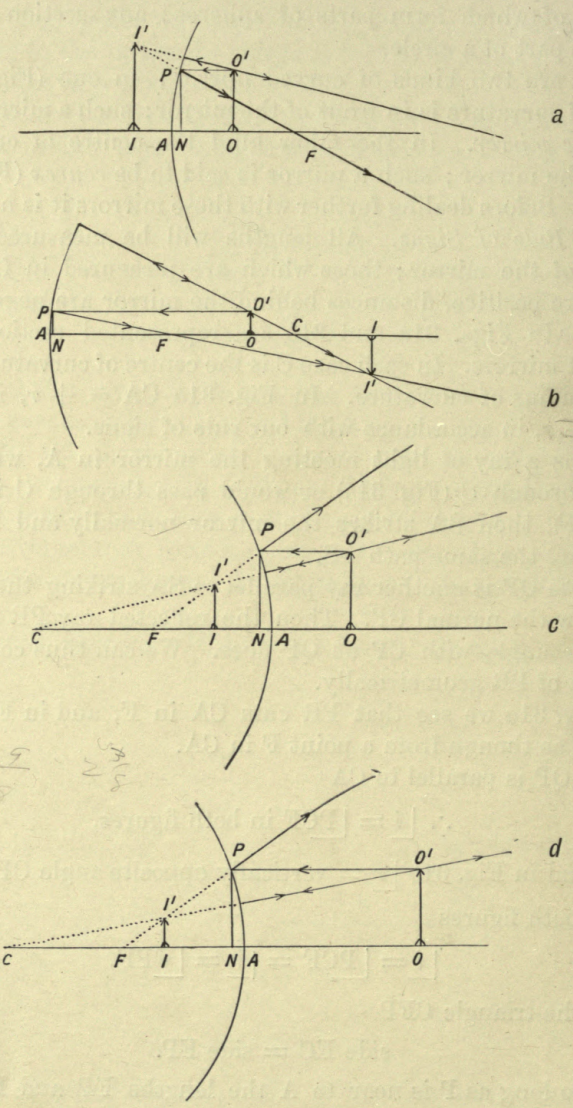



Fig. 32.

after reflection, either to or from a point nearly half-way between C and A.

Now the rays from a very distant object such as the sun are

practically parallel; consequently the image of a very distant object will lie half-way between C and A.

*The point where the image of a very distant object is formed by reflection in a mirror is called the Principal Focus, and the distance of this point from the mirror is called the Focal Distance or Focal Length of the mirror.*

Hence we have the rule that the principal focus of a mirror lies half-way between the centre of curvature and the surface of the mirror; or, the focal length of a mirror is equal to half its radius of curvature. We may express this thus in symbols:—

In the case of Fig. 31*a*. In the case of Fig. 31*b*.

$$+f = +\frac{r}{2} \qquad -f = -\frac{r}{2},$$

$$\text{or } f = \frac{r}{2} \text{ in both cases.}$$

§ 25.—Suppose that an object OO′ (Fig. 32) is placed in front of a curved mirror.

We know that a ray starting from O′ and passing through C, or going as though to or from C, is normal to the mirror, and is therefore reflected back along its former course. The image of O′ must therefore lie somewhere along O′C or O′C produced.

We know also that such a ray as O′P, parallel to CA, will, after reflection, pass through or appear to come from F. The image of O′ must therefore lie on PF or PF produced.

In Fig. 32*a* OO′ has been represented as fairly close to a concave mirror, and we see that PF and O′C meet at a point I′ behind the mirror. Rays from O′ will therefore, after reflection, proceed as though they came from I′, or I′ is a virtual image of O′ (*see* § 20).

In Fig. 32*b* OO′ has been drawn at some distance from a concave mirror; in this case I′ is found to be in front of the mirror. I′ is therefore a real image of O′.

In Figs. 32*c* and 32*d* OO′ is represented as being in front of a convex mirror. Here we find that, whatever the distance of OO′ from the mirror, I′ is always behind the mirror; a convex mirror can only give virtual images.

If a similar construction is gone through for other points on OO it will be found that the images of these points lie on II′ in every case.

II′ is therefore the image of OO′.

In order to arrive at a formula connecting the positions of OO′ and II′ we shall use the following symbols:—

Distance of object from mirror = OA = $u$.

Distance of image from mirror = IA = $v$.

According to the rule of signs the symbols assigned in the different figures will be—

In all figures OA = + $u$.

In Fig. 32$a$, 32$c$, 32$d$ IA = − $v$. In Fig. 32$b$ IA = + $v$.

In Fig. 32$a$, 32$b$ AF = + $f$. In Fig. 32$c$, 32$d$ AF = − $f$.

In all the figures draw PN perpendicular to AC.

The triangles I′IF and PNF are similar,

$$\therefore \frac{IF}{NF} = \frac{II'}{NP}.$$

The triangles II′C and OO′C are also similar,

$$\therefore \frac{IC}{OC} = \frac{II'}{OO'}.$$

But OO′ = PN, and N is so close to A that NF = AF approximately,

$$\therefore \frac{IF}{AF} = \frac{IC}{OC},$$

$$\text{or } IF \cdot OC = AF \cdot IC.$$

Substituting the values of $u$, $v$ and $f$, we have—

In Fig. 32$a$

$$\{f + (-v)\}\ \{r - u\} = f\{r + (-v)\},$$

$$\text{or } (f - v)\ (2f - u) = f(2f - v) \text{ since } r = 2f \text{ (§ 24)}.$$

In Fig. 32$b$

$$(v - f)\ (r - u) = f(v - r),$$

$$\text{or } (v - f)\ (u - 2f) = f(2f - v).$$

In Figs. 32$c$ and 32$d$

$$\{(-f) - (-v)\}\ \{u + (-r)\} = (-f)\ \{(-r) - (-v)\},$$

$$\text{or } (v - f)\ (u - r) = f(r - v).$$

In all cases on multiplying out

$$vu - 2vf - fu + 2f^2 = 2f^2 - fv,$$
$$\text{or } vu = fu + fv;$$

dividing through by $uvf$, we have

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}, \text{ the general formula for mirrors.}$$

§ 26. *Practical Proof of the Formulæ for a Concave Mirror.*—Set up a concave mirror on its stand as in Fig. 33 and place in front of it a pin supported by a clamp. The tip of the pin should be level with the centre* of face of the mirror. Put the pin about 20 to

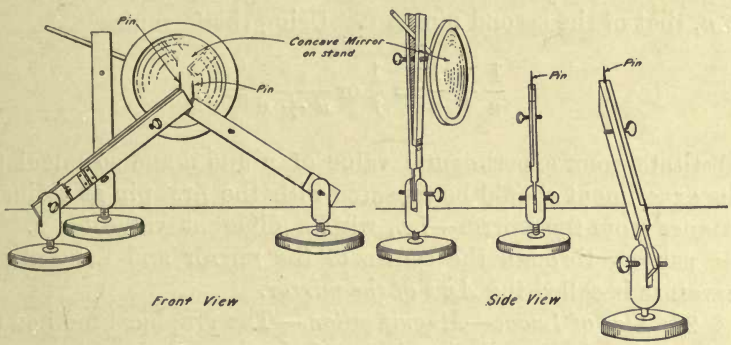



Fig. 33.

30 cms. distant from the mirror; on standing some way back it will be found possible to see an image of the pin upside down in front of the mirror. Examine this image by the principle of parallax (§ 19). If it is nearer to you than the pin, pull the pin towards you and continue doing so until the pin and its image appear together from whatever point they are viewed. Then, by the principle of parallax, the pin and its image are at the same distance from you and therefore at the same distance from the mirror.

Measure the distance of the pin from the mirror. Since the rays starting from the pin are reflected back to their point of origin, they must strike the mirror normally, and therefore must start from the centre of curvature. The distance measured is therefore the radius of curvature $= r$.

* This is often called the centre of the mirror; take care not to confuse it with the centre of curvature.

Somewhere between the pin and the mirror will be seen an inverted image of the room behind you and of objects outside the window. Choose the most distant of these objects (an object with a sharp outline, such as a chimney-pot, is preferable) and push the pin in towards the mirror until it coincides (by parallax) with the image of the chosen distant object. The distance of the pin from the mirror may now be measured and should be the focal distance $= f$.

It will be found that approximately $r = 2f$ (*see* § 24).

Place the pin at a point whose distance from the mirror lies between $f$ and $r$. Take a second pin and place it so that it coincides with the image of the first (*not* with its own image). Measure the distances of the pins from the mirror. The distance of the first pin $= u$, that of the second pin $= v$. Using the formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ or } \frac{uv}{u+v} = f,$$

substitute your experimental value of $v$ and $u$ and so calculate $f$. The experiment should be repeated with the first pin at a different distance from the mirror—*i.e.*, with a different value for $u$. The line passing through the centre of the mirror and the centre of curvature is called the *Axis of the mirror.*

§ 27. *Size of Image—Magnification.*—The graphical method used in § 25 for finding the position of the image also gives us the relative sizes of the object and image; and by drawing the whole diagram to scale a complete solution to the problem is possible. It is also possible to obtain a formula connecting the relative sizes of the object and image, since the triangles PNF and I'IF are similar, and therefore

$$\frac{\text{II}'}{\text{OO}'} = \frac{\text{II}'}{\text{PN}} = \frac{\text{FI}}{\text{FN}} = \frac{\text{FI}}{\text{FA}} = \frac{v-f}{f}.$$

$$\text{But } \frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{v-f}{vf}$$

$$\therefore \frac{\text{II}'}{\text{OO}'} = \frac{v-f}{vf} \times v = \frac{v}{u}.$$

This may also be more directly proved from Figs. 34*a*, 34*b* and 34*c*.

Any ray starting from O' will, after reflection, pass through I' or appear to come from I'.

Therefore the ray O'A will, after reflection, take the course AR where R is in AI' or I'A produced.

But $\angle O'AO = \angle RAI$ by the laws of reflection.

$\therefore$ triangles O'AO and I'AI are similar,

$$\therefore \frac{II'}{OO'} = \frac{IA}{OA} = \frac{v}{u}.$$

We see that in all cases the magnification, or linear ratio of the sizes of image and object, is equal to the ratio of their distances from the mirror.

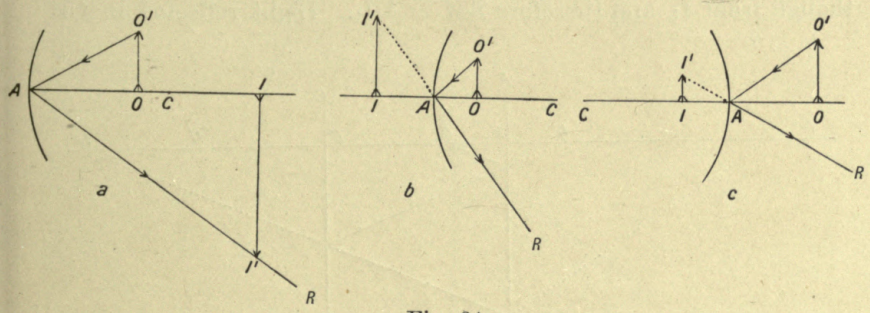



Fig. 34.

§ 28. *Various Cases Considered by means of the Formula.*—We may write the general formula for mirrors

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u - f}{uf}.$$

In the case of a *convex* mirror $f$ is negative, and we may write

$$V = \frac{U(-F)}{U-(-F)} = -\frac{UF}{U+F} = -F\frac{U}{U+F}.$$

In words, $V$ is always negative and less numerically than $F$; or the image formed by a convex mirror is always virtual and between the principal focus and the mirror.

In the case of a concave mirror $f$ is positive, and therefore

$$V = \frac{UF}{U-F} = F\frac{U}{U-F}.$$

(i.) If $U$ is greater than $F$, then $V$ is positive and greater than $F$.

(ii.) If $U$ is less than $F$, then $V$ is negative.

In words, when the object is further from a concave mirror than the principal focus the image is real and further from the mirror than the principal focus. When the object is nearer to a mirror than its principal focus the image is virtual and at a distance behind the mirror greater than the focal distance.

§ 29. *Combination of Mirrors.*—Let AB and CD be two plane mirrors (Fig. 35) placed parallel to one another and having an object O placed between them. There will be an image of O formed by reflection in AB at a point $I_1$ such that $ON = NI_1$.

The light reflected from AB will fall on CD and be reflected so as to appear to come from an image $I_2$. The light comes to CD as though from $I_1$, and therefore $I_2M = MI_1$. Light reflected in CD

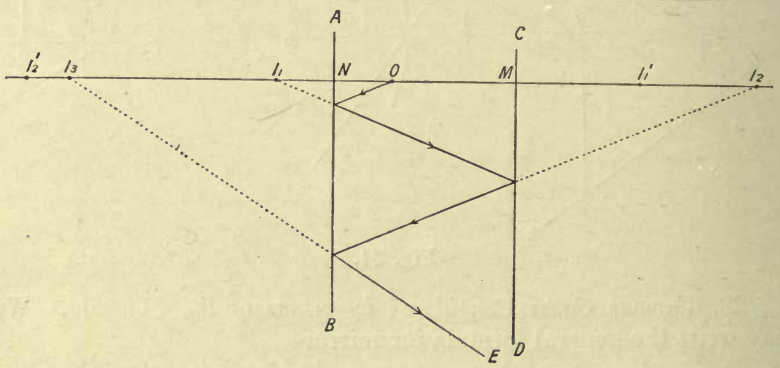



Fig. 35.

may go back to AB and be reflected as though it now came from $I_3$ where $I_3N = NI_2$. In this way an infinite series of images may be formed by alternate reflection in each mirror.

But light from O also falls on CD, and will, after reflection, proceed as though from an image $I_1'$ where $OM = I_1'M$. In this way we get a second series of images $I_1'$, $I_2'$, etc.

Suppose an eye placed at E looking at the image $I_3$. What is the path of the ray by which this eye sees $I_3$? The ray clearly comes to the eye as though from $I_3$. Therefore, by joining $EI_3$ we can get the path of the ray between AB and E. The ray comes to AB as though from $I_2$. Hence, by joining $I_2$ to the point where $EI_3$ cuts AB, we have the path of the ray from CD to AB. The ray comes to AB from O. We know where it leaves AB, and have only to join this point to O to complete the path of the ray.

§ 30.—Suppose that the mirrors, instead of being parallel, are inclined at an angle to one another. Let CA, CB (Fig. 36), be two such mirrors, and let O be an object between them.

Light from O falling on CA will be reflected as though it came from $I_1$ where $I_1N = NO$. The triangles CON and $CI_1N$ will therefore be equal, and therefore $CO = CI_1$. Hence $I_1$ will lie on a circle with radius CO and centre C. Similarly it may be shown that $I_2$, the image formed in CB by light from $I_1$, lies on the same circle,

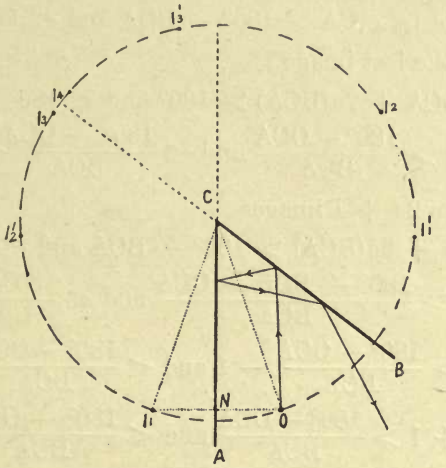



Fig. 36.

and that all the images lie on a circle with centre C and radius CO.

Fig. 36 is so drawn that light from $I_3$ can fall on CB, giving an image $I_4$, but light from $I_4$ cannot fall on CA. Hence $I_4$ is the last image formed by first reflection in CA.

Another set of images $I_1'$, $I_2'$, $I_3'$, can be formed by light falling first on CB.

$$
\begin{aligned}
\text{*Now angle } I_1CA &= \text{angle } OCA,\\
\text{and angle } I_2CB &= \text{angle } I_1CB\\
&= \angle I_1CA + \angle ACB\\
&= \angle OCA + \angle ACB\\
\therefore \text{angle } I_2CA &= \angle OCA + \text{twice } \angle ACB,\\
\text{similarly angle } I_3CA &= \angle I_2CA\\
&= \angle OCA + \text{twice } \angle BCA.
\end{aligned}
$$

* This mathematical discussion is not of great importance in itself, but illustrates a method of solution which may be applied to similar problems.

Or, in general, the angle between the $2n^{th}$ image and the mirror CA = the angle between the $(2n + 1)^{th}$ image and CA,

$$\text{or } I_{2n}CA = OCA + 2n\,(BCA).$$

It will be impossible for light from the image $I_{2n}$ to reach CA, or from the image $I_{2n+1}$ to reach CB if $I_{2n}$ or $I_{2n+1}$ lie (like $I_4$, Fig. 36) between AC and BC produced. That is, the image $I_{2n}$ will be the last of its series if

$$\text{angle } I_{2n}CA > 180° \text{ and } < 180° + BCA,$$
$$\text{or if angle } I_{2n+1}CA > 180° - BCA \text{ and } < 180°.$$

Therefore for the last image $I_{2n}$

$$\text{angle } OCA + 2n(BCA) > 180° \text{ and } < 180° + BCA,$$
$$\text{or } 2n > \frac{180° - OCA}{BCA} \text{ and } < \frac{180° - OCA}{BCA} + 1,$$

and if there are $2n + 1$ images

$$OCA + 2n(BCA) > 180° - BCA \text{ and } < 180°,$$
$$\text{or } 2n > \frac{180° - BCA - OCA}{BCA} \text{ and } < \frac{180° - OCA}{BCA}$$
$$> \frac{180° - OCA}{BCA} - 1 \text{ and } < \frac{180° - OCA}{BCA},$$
$$i.e., 2n + 1 > \frac{180° - OCA}{BCA} \text{ and } < \frac{180° - OCA}{BCA} + 1.$$

In both cases the limits are the same.

If the reflection takes place first in the mirror CB, then the number of images will be N′ where

$$N' > \frac{180° - OCB}{BCA} \text{ and } < \frac{180° - OCB}{BCA} + 1.$$

The total number of images is therefore given by

$$N + N' > \frac{180° - \angle OCA}{\angle BCA} + \frac{180° - \angle OCB}{\angle BCA} \text{ and}$$
$$< \frac{180° - \angle OCA}{\angle BCA} + \frac{180° - \angle OCB}{\angle BCA} + 2,$$
$$i.e., N + N' > \frac{360°}{\angle BCA} - 1$$
$$< \frac{360°}{\angle BCA} + 1.$$

Suppose, for instance, that $\angle BCA = 60°$, then

$$N + N' > 6 - 1 \text{ or } > 5$$
$$< 6 + 1 \text{ or } < 7.$$

There would clearly be six images. If, however, $\angle BCA = 55°$,

$$N + N' > 5{\cdot}5 \text{ and } < 7{\cdot}5.$$

There may therefore be either six or seven images according to the position of the object.

A few cases are given in the following table:—

| $\angle BCA$ . . | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|
| No. of images . | 36* | 18* | 12* | 9* | 7 or 8 | 6* | 5 or 6 | 4 or 5 | 4* |

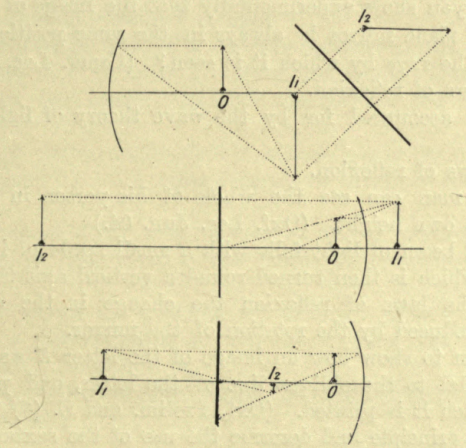



Fig. 37.

If $\dfrac{360°}{\angle CBA}$ is a whole number, then since

$$\angle I_{2n}CA = \angle OCA + 2n\angle BCA$$
$$\text{and } \angle I'_{2n}CB = \angle OCB + 2n'\angle BCA,$$
$$\text{or } \angle I'_{2n}CA = \angle OCB + 2n'\angle BCA - \angle BCA,$$
$$\therefore \angle I_{2n}CA + \angle I'_{2n}CA = \angle OCA + \angle OCB + (2n + 2n' - 1)\angle BCA$$
$$= (2n + 2n')\angle BCA$$
$$= 360°,$$

or the last image of one series ($I_{2n}$) coincides with the last of the other series ($I'_{2n}$). One will therefore actually see one less image than is indicated in the table. Such cases are marked with an asterisk.

§ 31.—Combinations of a plane and a curved mirror offer no difficulties if it is remembered that the image formed by the first reflecting surface serves as an object for the second reflection. One or two examples are shown solved graphically in Fig. 37. Here O is the object, $I_1$ the first image and $I_2$ the last image. Other images (not shown) could be formed by further reflections.

*Examples.*

1. Give the laws of reflexion of light, and describe how you would proceed to test them experimentally. (Oxf. Prelim. and C. (3) 04.)

2. How would you show experimentally that the image of an object seen by reflexion in a plane mirror is always in the same position, whatever be the direction of the rays by which it is seen? (Camb. Loc. Jun.)

3. State the laws of reflexion.

How are they accounted for by the wave theory of light? (Oxf. Loc. Sen. 05.)

4. State the laws of reflexion.

Prove that a man can see the whole of his person in a mirror whose length is half his own height. (Oxf. Loc. Jun. 04.)

5. A horizontal beam of light falls with a small angle of incidence $i$ on a vertical mirror, which is then turned round a vertical axis through an angle $a$. Find from the laws of reflexion the change in the direction of the reflected beam produced by the rotation of the mirror.

Make a diagram to show how an image of the letter $Z$ would be formed in a mirror parallel to the oblique line of the letter, and perpendicular to the paper on which it is printed. (Oxf. Prelim. and C. (3) 08.)

6. Explain the principle and describe the use of the sextant. (Army 00.)

7. How would you arrange two mirrors so as to be able to see the side of your head when looking straight forward? Give a drawing showing the complete course of a ray. (Lond. Inter. Sci. 02.)

8. Explain the action of the kaleidoscope.

How many images can be seen when a candle is placed between two plane mirrors which meet at an angle of 30 degrees? (Camb. M.B. 05.)

9. Explain carefully the difference between a real and a virtual image.

Describe briefly some method of measuring the speed of propagation of light. (Oxf. Loc. Jun. 04.)

10. Define *principal focus* in the case of a spherical mirror, and show that the radius of the mirror is twice the focal length.

Find the position of the image of a small object placed 5 ins. in front of a concave mirror of radius 1 ft. (Oxf. Prelim. and C. (3) 04.)

11. Explain how the position of an image of an object formed by reflexion at a curved surface may be found graphically. Give diagrams for three different cases. (Camb. Spec. 05.)

12. Rays fall on a polished concave hemisphere parallel to the axis; give a construction for the reflected rays, and draw two reflected rays corresponding to an incident ray near the axis and to one about half the radius distant from the axis.

How does a parabolic mirror behave with respect to rays parallel to its axis? (Oxf. Loc. Sen. 07.)

13. Describe any two experimental methods for determining the radius of curvature of a concave spherical mirror. (Oxf. Loc. Jun. 07.)

14. Explain, with diagrams, in what circumstances there can be formed by reflexion an image (*a*) equal to, (*b*) half the size of, the object. Indicate in each case whether the image is a real or a virtual one. (Camb. Loc. Jun.)

15. What do you understand by conjugate foci? Determine graphically the size of the image of an object 1 cm. long placed perpendicularly to the axis of a concave mirror whose radius is 20 cms. The object is 15 cms. from the mirror. (Camb. Loc. Jun. 07.)

16. State the laws of reflexion of light.

A candle is placed in front of a concave spherical mirror of radius 6 ft., and at a distance of 9 ft. from it. Draw the course of at least three rays from a point on the candle, and find the position and size of the image. (Matric. 04.)

17. Under what circumstances does a concave mirror form (1) a real image, (2) a virtual image? Draw a clear diagram to illustrate the formation of the latter.

If an object placed 6 ins. from a concave mirror produces a virtual image at a distance of 12 ins., what is the radius of curvature of the mirror? (Army 03.)

18. A concave mirror has a radius of 30 cms. Where must the object be placed so that the magnification may be 3, when the image is (*a*) real (*b*) virtual? (Camb. Loc. Jun. 06.)

19. How would you find experimentally the focal length of a convex mirror? (Camb. Loc. Sen. 07.)

## CHAPTER III

### REFRACTION

§ 32. *Laws.*—When light reaches the surface between two transparent media, part of the beam may be reflected, but part may pass into the second medium, and its direction will usually be altered. This is generally expressed by saying that the beam has been bent or refracted. If a stick is partly immersed in water and viewed from the side it will appear bent because the light from the lower end of the stick is bent in crossing the surface of the water (Fig. 38). In Fig. 39 a section of the stick and water taken at

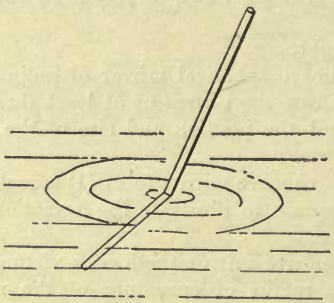


Fig. 38.

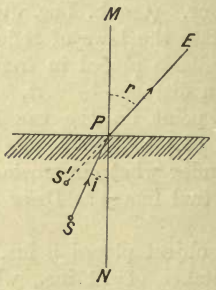



Fig. 39.

right angles to Fig. 38 is shown. The stick, which is really at S, appears to be at S′. A ray coming to an eye at E must therefore have been bent downwards in coming through the boundary water-air, and must have taken the course SPE. If the normal NPM is drawn, the angle SPN is called the *angle of incidence* and the angle EPM is the *angle of refraction*. The first law of refraction is that the angles of incidence and refraction lie in one plane, the second law was found by Snell and states that *the sine of the angle of incidence is always the same multiple of the sine of the angle of refraction* and that this multiple is a characteristic of a boundary. The ratio $\frac{\sin i}{\sin r}$ (*see* Appendix II., § 1) is called the *refractive index* of the boundary. Thus in the case of the boundary water-air the refractive index $= \frac{\sin \text{SPN}}{\sin \text{EPM}}$.

In Fig. 40 is shown a ray QPR passing from air to water at the point P; in this case

$$\sin i = \sin \text{QPN} = \frac{\text{QN}}{\text{QP}},$$

$$\sin r = \sin \text{RPM} = \frac{\text{RM}}{\text{RP}}$$

$$\therefore \text{ refractive index} = \mu = \frac{\sin i}{\sin r} = \frac{\text{QN}}{\text{QP}} \times \frac{\text{RP}}{\text{RM}}.$$

$$\text{If we make QP} = \text{RP, then } \mu = \frac{\text{QN}}{\text{RM}}.$$

If the course of one ray is known before and after refraction this gives an easy way of drawing the paths of other rays.

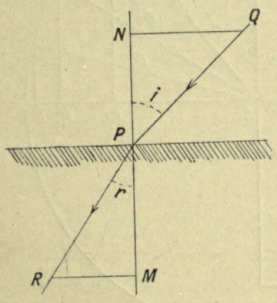



Fig. 40.

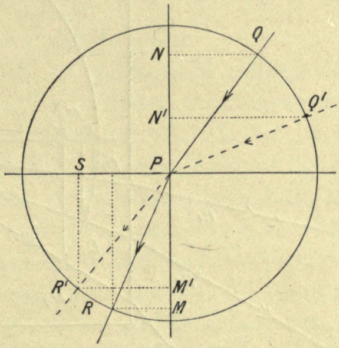



Fig. 41.

Suppose the course of the ray QPR (Fig. 41) is given. Draw the normal at P, and with P as centre, describe a circle cutting the ray in Q and R. Then PQ = PR, and therefore $\mu = \frac{\text{QN}}{\text{RM}}$.

Now let Q'P be some other ray incident at P and cutting the circle in Q'. Draw Q'N' perpendicular to NP and cut off PS so that $\frac{\text{Q'N'}}{\text{PS}} = \frac{\text{QN}}{\text{RM}} = \mu$.

From S draw SR' perpendicular to PS and cutting the circle in R'. Then since R'M' = SP,

$$\therefore \frac{\text{Q'N'}}{\text{R'M'}} = \frac{\text{QM}}{\text{RM}} = \mu,$$

and PR' is the course of the ray Q'P after refraction.

§ 33. *Experimental Proof.*—Lay a rectangular block of glass such as is sold as a paperweight on a sheet of paper and draw its outline carefully with a pencil. Choose any point A in one of the longer sides and draw lines such as $P_1A$, $P_2A$, $P_3A$, converging to A. Replace the block and stick pins $P_1$, $Q_1$, into points on the line $P_1A$. Now stick in pins $R_1$, $S_1$, so that the four pins $P_1$ $Q_1$ $R_1$ $S_1$ appear in line. In the same way the pins $R_2$ $S_2$ are put in to appear in line with $P_2$ $Q_2$, and $R_3$ $S_3$ to appear in line with $P_3$ $Q_3$.

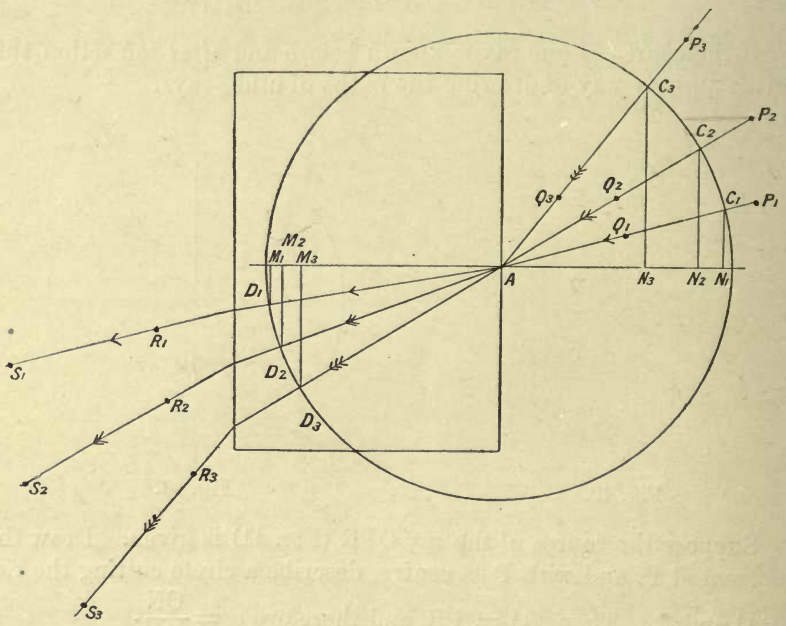



Fig. 42.

Remove the block and draw lines joining $R_1$ $S_1$, $R_2$ $S_2$, $R_3$ $S_3$, and produce them to the nearer edge of the block; by joining the points so found to A the paths of the rays inside the block can be drawn.

With A as centre describe a circle, and from the points where this cuts the rays drop perpendiculars on the normal through A.

In the case of the ray $P_1Q_1R_1S_1$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin P_1AN_1}{\sin D_1AM_1} = \frac{C_1N_1}{C_1A} \div \frac{D_1M_1}{D_1A} = \frac{C_1N_1}{D_1M_1}.$$

On measuring the lengths $D_1M_1$, $D_2M_2$, $D_3M_3$, $C_1N_1$, $C_2N_2$ and $C_3N_3$, the ratios $\frac{C_1N_1}{D_1M_1}$, $\frac{C_2N_2}{D_2M_2}$, $\frac{C_3N_3}{D_3M_3}$, can be calculated. It will be found that they are equal, and the value so obtained is called the Refractive Index from air to glass.

Further, it will be found that the rays $R_1S_1$, $R_2S_2$, $R_3S_3$, are parallel to $P_1Q_1$, $P_2Q_2$, $P_3Q_3$, respectively. Since the faces of the glass block are parallel, the normals at the points of emergence of the rays will all be parallel to the normal at A. Hence the angles of incidence at the second face are equal to the angles of refraction at the first face, and the angles of emergence (angles of refraction) at the second face are equal to the angles of incidence at the first face.

Hence $\mu$ from air to glass $= \frac{1}{\mu}$ from glass to air. If we examine the matter theoretically, the reason for this is plain.

§ 34. *Theoretical.*—Let AB (Fig. 43) be the front of a plane wave arriving at PQ, the surface of separation between two transparent media, and let BA′ be a ray perpendicular to AB. Let $t$ be the time taken by the light from B to reach A′, $v$ the velocity of light in the first medium, and $v'$ its velocity in the second medium, so that $BA' = vt$. Draw AC parallel to BA′ and A′C parallel to BA. Then A′C is the position which the wave-front would have occupied at time $t$ had it travelled on unobstructed into the second medium. But as each point of the wave-front arrives at PQ it becomes the origin of a secondary spherical wave in the second medium which is propagated with velocity $v'$. Therefore the refracted wave at A will in time $t$ have developed into a sphere of radius $v't$. With centre A and radius $= v't$ describe a sphere, and let A′B′ represent a tangent plane to this sphere through A′. Then A′B′ shall represent the front of the refracted wave at the instant when the light from B reaches A′.

For take any point R in AB and draw RS parallel to BA′ and SR′ parallel to AB′. Now light from R will travel over the distance RS with velocity $v$, and will then develop a spherical wave in the second medium with velocity $v'$, and we have to show that in $t$ seconds after it leaves R this spherical wave will touch A′B′.

Let $T$ be the time it travels in the first medium and $T'$ the time in the second medium. Now $T = \frac{RS}{v}$ and by similar triangles

$$\frac{RS}{AS} = \frac{BA'}{AA'} = \frac{vt}{AA'} \therefore \frac{RS}{v} = T = \frac{AS \times t}{AA'}.$$

From the similar triangles A′AB′ and A′SR′ it follows in like manner that

$$T'' = \frac{t \times \text{A'S}}{\text{AA'}}$$

$$\therefore T' + T'' = \frac{t.\text{A'S}}{\text{AA'}} + \frac{t.\text{AS}}{\text{AA'}}$$

$$= t\frac{\text{A'S} + \text{AS}}{\text{AA'}} = t$$

$\therefore$ the light from R will arrive at R′ in the time $t$.

But R may be any point on the incident wave-front AB. Hence A′B′ is the position of the refracted wave-front at the instant when

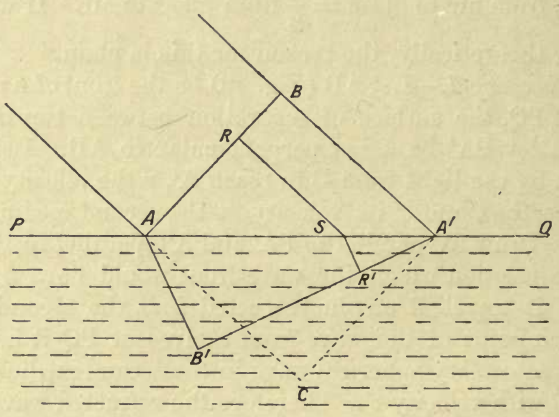



Fig. 43.

the light from B has reached A′. Now the angle BAA′ = the angle of incidence = $i$, say; and the angle B′A′A = the angle of refraction = $r$, say.

$$\therefore \frac{\sin i}{\sin r} = \frac{\text{A'B}}{\text{AA'}} \div \frac{\text{AB'}}{\text{AA'}} = \frac{\text{A'B}}{\text{AB'}} = \frac{vt}{v't} = \frac{v}{v'}.$$

Hence the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, which proves that Snell's law of refraction is consistent with the wave theory.

We learn from the above that the refractive index of a medium B with regard to another medium A is the ratio of the velocity of light in A to its velocity in B.

Moreover, since

$$\mu \text{ air to glass} = \frac{\text{velocity in air}}{\text{velocity in glass}}$$

$$\text{and } \mu \text{ glass to air} = \frac{\text{velocity in glass}}{\text{velocity in air}}$$

$$\therefore \mu \text{ air to glass} = \frac{1}{\mu \text{ glass to air}},$$

a result which we found to be true experimentally.

§ 35. *Apparent Position of Object viewed through a Block of Glass.*—When an object such as O (Fig. 44) is viewed through a block of glass it appears nearer than it really is. The ray ONM being

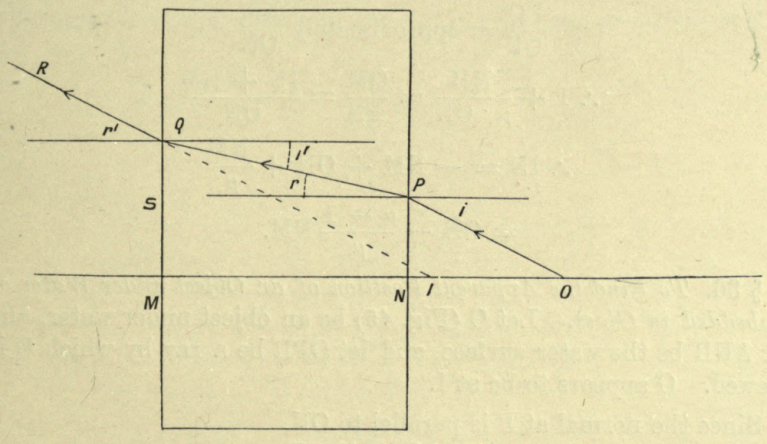



Fig. 44.

normal is not deviated, but the ray OPQR is bent both at P and Q, and though QR is parallel to OP, yet the ray is so displaced that QR appears to come from I. And therefore the object O appears to be at I.

The normals at P and Q are parallel to OM, and QR is parallel to OP; hence

$$\angle i \text{ at P} = \angle \text{PON},$$
$$\angle r \text{ at P} = \angle i' \text{ at Q},$$
$$\angle r' \text{ at Q} = \angle \text{QIM} = \angle i \text{ at P} = \angle \text{PON},$$

and the triangles QIM, PON, are similar.

$$\therefore \frac{\text{QM}}{\text{PN}} = \frac{\text{IM}}{\text{ON}} = \frac{\text{IN} + \text{NM}}{\text{ON}},$$

$$\text{but } \mu = \frac{\sin i}{\sin r} = \frac{\sin \text{PON}}{\sin \text{QPS}} = \frac{\text{PN}}{\text{OP}} \div \frac{\text{QS}}{\text{PQ}}$$

$$\therefore \text{QS} = \frac{\text{PN}}{\text{OP}} \frac{\text{PQ}}{\mu};$$

and since QM = SM + QS

$$\therefore \text{QM} = \text{PN} + \frac{\text{PN} \cdot \text{PQ}}{\text{OP} \cdot \mu},$$

$$\text{or } \frac{\text{QM}}{\text{PN}} = 1 + \frac{\text{PQ}}{\mu \cdot \text{OP}}.$$

Now the bundle of rays by which an eye sees an object is small, and we may therefore put

$$\frac{\text{PQ}}{\text{OP}} = \text{approximately } \frac{\text{NM}}{\text{ON}}$$

$$\therefore 1 + \frac{\text{NM}}{\mu \cdot \text{ON}} = \frac{\text{QM}}{\text{PN}} = \frac{\text{IN} + \text{NM}}{\text{ON}}$$

$$\therefore \text{IN} = -\text{NM} + \text{ON} + \frac{\text{NM}}{\mu}$$

$$= \text{ON} - \frac{\mu - 1}{\mu} \text{NM}.$$

§ 36. *To Find the Apparent Position of an Object under Water or Imbedded in Glass.*— Let O (Fig. 45) be an object under water, and let ANB be the water surface, and let OPR be a ray by which O is viewed. O appears to be at I.

Since the normal at P is parallel to ON,

$$\therefore \angle i \text{ at P} = \angle \text{NOP},$$

$$\angle r \text{ at P} = \angle \text{NIP}$$

$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin \text{NOP}}{\sin \text{NIP}} = \frac{\text{NP}}{\text{PO}} \div \frac{\text{NP}}{\text{PI}} = \frac{\text{PI}}{\text{PO}}.$$

So long as P is near to N we may write approximately

$$\frac{\text{PI}}{\text{PO}} = \frac{\text{NI}}{\text{NO}}$$

$$\therefore \text{NI} = \mu \text{ NO approximately.}$$

§ 37. *Verification of the above Formulæ.*—(i.) Take a microscope which is in focus when the object is about 5 cms. below the lower lens. Focus this on a sheet of paper. Lay a block of glass on the paper and screw up the microscope until the paper is again in focus. The distance through which the microscope is screwed is OI

(Fig. 45) ; suppose this is 1 cm. Continue screwing up the microscope until the top surface of the glass is focussed. The total distance through which the microscope has been raised will be the thickness of the glass ; suppose this is 3 cms.

Then NI = ON − OI = 3 − 1 = 2 cms.

Since one surface of the glass is in close contact with the paper the conditions are those of § 36.

$$\therefore \mu \text{ from glass to air} = \frac{\text{NI}}{\text{NO}} = \frac{2}{3},$$

$$\text{or } \mu \text{ from air to glass} = \frac{3}{2} = 1{\cdot}5.$$

(ii.) Raise the block about a couple of centimetres from the paper, and clamp it. Screw up the microscope until the upper surface of the

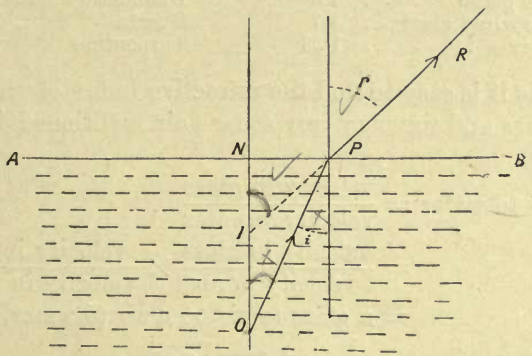



Fig. 45.

glass is again in focus. The distance through which the microscope is raised = distance through which glass block is raised,
= distance from lower surface of block to paper,
= ON (Fig. 44).
Suppose this is 2·5 cms.

Screw the microscope down until the paper is again in focus; the distance through which the microscope is lowered = IM; let this be 4·5 cms.

Then IN = IM − NM = 4·5 − 3 = 1·5,

$$\therefore 1{\cdot}5 = 2{\cdot}5 - \frac{\mu - 1}{\mu} 3,$$

$$\text{whence } \mu = \frac{1}{1 - \frac{2{\cdot}5 - 1{\cdot}5}{3}} = \frac{3}{2} = 1{\cdot}5.$$

§ 38. *Value of the Refractive Index.*—It was mentioned in § 32 that the refractive index is characteristic of a given boundary, and in § 34 it was shown that

$$\mu \text{ from A to B} = \frac{\text{velocity of light in A}}{\text{velocity of light in B}}.$$

Now the commonest cases of refraction involve passage of light from air to another medium, so that we may write

$$\mu \text{ from air to B} = \frac{\text{velocity in air}}{\text{velocity in B}}.$$

The value of $\mu$ therefore involves the value of the velocity in B and will naturally depend on the nature of B. The values of $\mu$ from air to some ordinary media are given in the following table :—

| | | | |
|---|---|---|---|
| Plate glass . . | 1·5 | Diamond . . | 2·47 |
| Heavy lead glass . | 1·7 | Quartz . . | 1·55 |
| Water . . . | 1·33 | Turpentine . | 1·45 |

From this it is easy to find the refractive index of the boundary between glass and water or any other pair mentioned in the table. For instance,

$$\mu \text{ glass to water} = \frac{\text{velocity in glass}}{\text{velocity in water}},$$

$$= \frac{\text{velocity in glass}}{\text{velocity in air}} \times \frac{\text{velocity in air}}{\text{velocity in water}},$$

$$= \mu \text{ glass to air} \times \mu \text{ air to water},$$

$$= \frac{1}{1{\cdot}5} \times 1{\cdot}33 = 0{\cdot}89.$$

§ 39. *Change in Angle of Refraction produced by Change in Angle of Incidence.*—We know that when the angle of incidence and the refractive index are known then it is possible to calculate the angle of refraction from the formula $\frac{\sin i}{\mu} = \sin r$. Suppose, for instance, that we are dealing with a ray passing from air to water; then $\sin r = \frac{\sin i}{1{\cdot}33}$; from this formula we can construct the following table :—

| $i$ | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|
| $\sin i$ | ·1736 | ·3420 | ·5000 | ·6428 | ·7660 | ·8660 | ·9397 | ·9848 | 1·000 |
| $\sin r$ | ·1305 | ·2572 | ·3759 | ·4833 | ·5760 | ·6512 | ·7065 | ·7405 | ·7518 |
| $r$ | 7° 30′ | 14° 54′ | 22° 5′ | 28° 54′ | 35° 10′ | 40° 38′ | 44° 57′ | 47° 46′ | 48° 45′ |

From such a table we can learn two important facts:—

(i.) $r$ does not increase so rapidly as $i$. When $i$ increases by 10° $r$ increases by amounts varying from 7° 30′ to 1°, but always by an amount less than 10°.

(ii.) When the incident ray grazes the surface, *i.e.*, when $i = 90°$ the value of $r$ is only 48° 45′.

Now suppose that the ray, instead of passing from air to water, were passing from water to air. Then we have

$$\frac{\sin i}{\mu \text{ from water to air}} = \sin r,$$

$$\text{or } \sin i \times \mu \text{ from air to water} = \sin r;$$

in other words, the values of $i$ and $r$ in the above table are reversed.

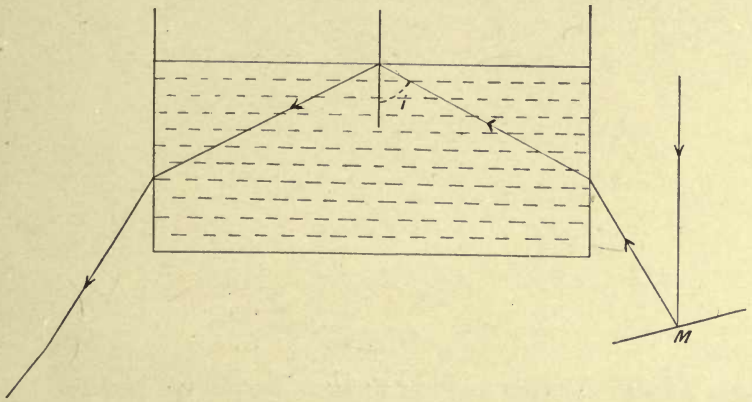



Fig. 46.

When $i = 28° 54'$ then $r = 40°$, and when $i = 48° 45'$ then $r = 90°$, or the emergent ray grazes the surface.

Suppose that $i$ now becomes 50°, then $\sin i = \cdot 7660$, and therefore $\sin r = 1 \cdot 33 \times \cdot 7660 = 1 \cdot 019$; but *there is no angle whose sine is greater than* 1, and consequently refraction in accordance with Snell's law is impossible, and the ray is totally reflected.

§ 40. *Total Reflection—Critical Angle.*—This is only possible when the media are such that $i < r$, that is, when $\mu < 1$, as, for instance, when light is passing from water or from glass to air. If light is passing from air to glass, $\mu > 1$, and therefore $\sin i > \sin r$, and $r$ is always less than 90°.

The phenomenon may be easily seen by means of a trough such as is shown in Fig. 46. A beam of light from a lantern is allowed

to fall on a plane mirror M, which can be so tilted as to throw the beam up to the surface of the water. So long as the angle of incidence ($i$) on the surface is less than 48° 45′ the beam will emerge into the air above the water; but when $i > 48° 45'$ the beam is totally reflected, and none emerges through the surface. (The experiment shows well if the water is tinted with an alkaline solution of fluorescein or of eosin.)

The value of $i$ for which the beam just emerges (for water

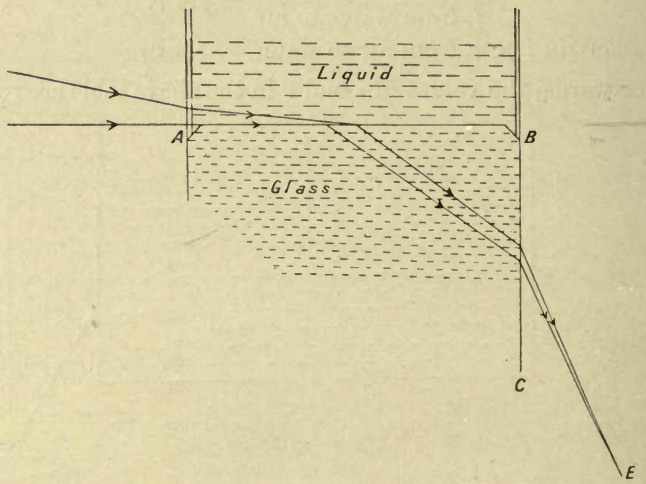



Fig. 47.

48° 45′) is called the *Critical Angle* of the medium. The critical angle is readily calculated when $\mu$ is known. Let it be $\phi$.

Then $\sin i = \mu \sin r$,

$\therefore \sin \phi = \mu \sin 90°$,

or $\sin \phi = \mu$ since $\sin 90° = 1$.

The critical angles from various media to air are given in the following table:—

| Medium | $\sin \phi$. | $\phi$. |
|---|---|---|
| Ordinary glass . . . | ·6667 | 41° 49′ |
| Water . . . . . | ·7518 | 48° 45′ |
| Diamond . . . . . | ·4132 | 24° 24′ |
| Quartz . . . . . | ·6452 | 40° 11′ |

Notice the extremely small value of $\phi$ for diamond; this is in a large measure the cause of its brilliant sparkle.

§ 41. An observation of the critical angle often proves an easy way of determining $\mu$. The *Pulfrich Refractometer* for liquids depends on this principle. The liquid is placed in the vessel shown in section above the line AB in Fig. 47. A convergent beam is sent along the surface AB and is refracted into the glass prism

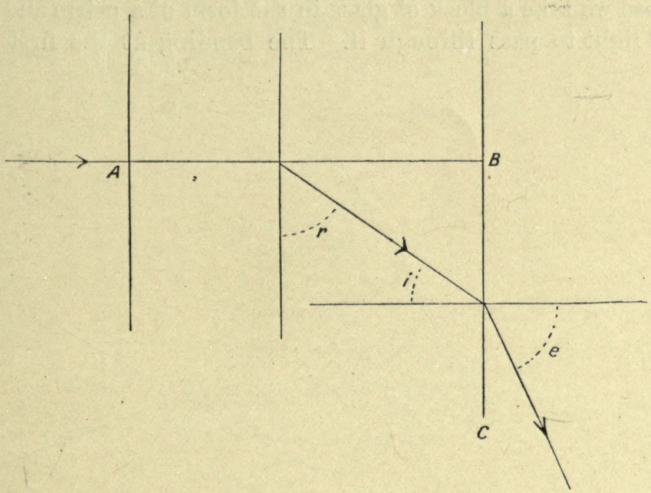



Fig. 48.

ABC and out into the air through the face CB, and can be viewed by an eye at E.

Arrangements are made for observing the angle of emergence ($e$, Fig. 48) of the last ray to pass the boundary AB.

Then we know that if $\mu_{ag}$ is the refractive index from air to glass

$$\frac{sin\ e}{sin\ i} = \mu_{ag} \text{ (Fig. 48).}$$

And if $\mu_{lg}$ is the refractive index from liquid to glass

$$\frac{sin\ 90°}{sin\ r} = \mu_{lg}, \text{ or } sin\ r = \frac{1}{\mu_{lg}}.$$

But $r = 90° - i$, and therefore $sin\ r = cos\ i$.

Moreover, $\mu_{lg} = \mu_{ag} \div \mu_{al}$ (*see* § 38),

$$\therefore \mu_{al} = \frac{\mu_{ag}}{\mu_{lg}} = \mu_{ag} \sin r = \mu_{ag} \cos i$$
$$= \mu_{ag}\sqrt{1 - \sin^2 i} = \mu_{ag}\sqrt{1 - \frac{\sin^2 e}{\mu^2_{ag}}}$$
$$= \sqrt{\mu^2_{ag} - \sin^2 e}.$$

Since $\mu_{ag}$ is known and $e$ can be observed, $\mu_{al}$ is readily calculated.

§ 42. *Refraction through a Block whose Sides are not Parallel.*— Suppose we take a block of glass in the form of a prism and allow a ray of light to pass through it. The bending at the first surface

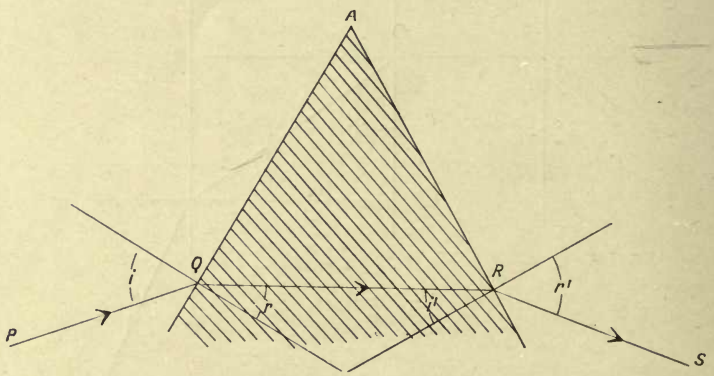



Fig. 49.

can readily be determined graphically by the method of § 32. We thus construct the path PQR of a ray incident at Q (Fig. 49).

On meeting the second surface at R a similar construction will give us the path RS of the emergent ray, remembering that $\mu$ glass to air $= 1 \div \mu$ air to glass.

We see that on the whole the ray is bent away from the angle A of the prism.

Such a ray can be traced with pins just as was done for a ray passing through a rectangular block in § 33.

Pins are stuck into points in the line PQ, and other pins are stuck in so as to appear in line with the first two pins when viewed through the prism. The line RS is thus obtained, and QR can be joined.

By cutting off equal lengths from Q along QP and QR and dropping perpendiculars on the normal at Q the refractive index of the

material of the prism can be obtained just as was done in the case of the rectangular block.

In Fig. 50 the ray PQRS is shown, but the prism is omitted. Had there been no prism the ray PQ would have continued along PQT. It has, however, been bent into the direction RS, *i.e.*,

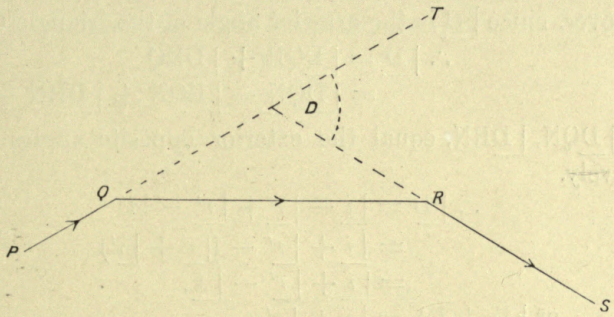



Fig. 50.

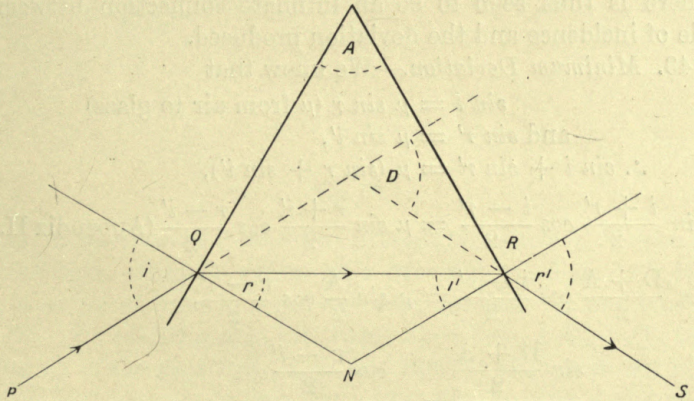



Fig. 51.

bent through the angle D. This angle D is called the *Angle of Deviation*.

In Fig. 51 the ray PQRS, the angle of deviation D, the normals to the prism faces QN and RN, and the prism faces QA and RA are all shown.

Since the figure AQNR is a quadrilateral

$$\therefore \angle QAR + \angle ARN + \angle RNQ + \angle AQN = 360°.$$

But $\angle AQN$ and $\angle ARN$ are right angles by construction,

$$\therefore \angle QAR + \angle RNQ = 180°.$$

In the triangle QRN the sum of the angles

$$\angle QRN + \angle RNQ + \angle RQN = 180°,$$

$$\therefore \angle QAR = \angle RQN + \angle QRN,$$

$$i.e. \ \angle A = \angle r + \angle i'.$$

Moreover, since $\angle D$ is the exterior angle of the triangle QDR,

$$\therefore \angle D = \angle DQR + \angle DRQ$$

$$= \angle DQN - \angle RQN + \angle DRN - \angle QRN.$$

Now $\angle DQN$, $\angle DRN$, equal the exterior opposite angles $i$ and $r'$ respectively.

$$\therefore \angle D = \angle i - \angle r + \angle r' - \angle i'$$

$$= \angle i + \angle r' - (\angle r + \angle i')$$

$$= \angle i + \angle r' - \angle A,$$

$$\text{or } \angle D + \angle A = \angle i + \angle r'.$$

There is thus seen to be an intimate connection between the angle of incidence and the deviation produced.

§ 43. *Minimum Deviation.*—We know that

$$\sin i = \mu \sin r \ (\mu \text{ from air to glass})$$

$$\text{and } \sin r' = \mu \sin i',$$

$$\therefore \sin i + \sin r' = \mu (\sin r + \sin i'),$$

$$\text{or } \sin \frac{i + r'}{2} \cos \frac{i - r'}{2} = \mu \sin \frac{r + i'}{2} \cos \frac{r - i'}{2} \text{ (Appendix II., § 8),}$$

$$\text{or } \sin \frac{D + A}{2} \cos \frac{i - r'}{2} = \mu \sin \frac{A}{2} \cos \frac{r - i'}{2},$$

$$\text{or } \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}} = \mu \frac{\cos \frac{r - i'}{2}}{\cos \frac{i - r'}{2}}.$$

It was pointed out in § 39 (i.) that when $i > r'$ then $r > i'$, and that $(i - r')$ is always greater than $(r - i')$, $\therefore \cos \frac{i - r'}{2}$ is always less than $\cos \frac{r - i'}{2}$. But if $r = i'$ then $i = r'$, and

$$\frac{\cos \frac{r - i'}{2}}{\cos \frac{i - r'}{2}} = \frac{\cos 0°}{\cos 0°} = \frac{1}{1} = 1.$$

The expression

$$\frac{\cos\dfrac{r - i'}{2}}{\cos\dfrac{i - r'}{2}}$$

therefore has its least value when $i = r'$; *i.e.*, when the passage of the ray is symmetrical; and hence D, the deviation, is least when $i = r'$.

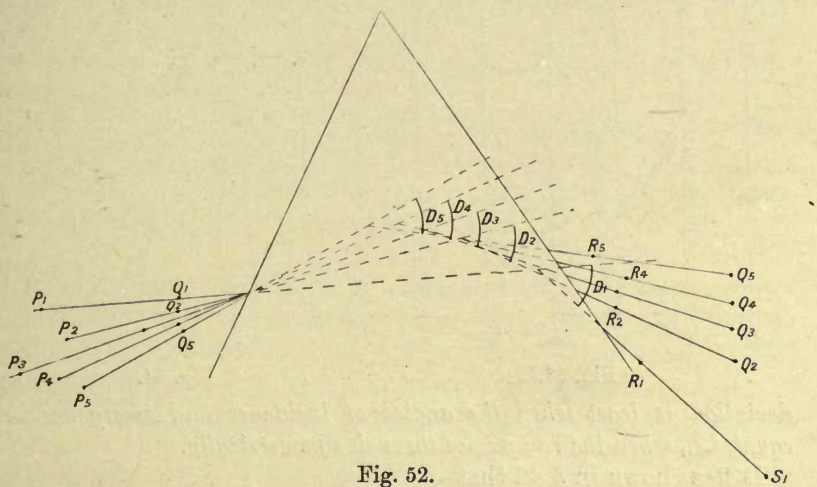



Fig. 52.

§ 44. *Experimental Connection between the Angle of Incidence and the Deviation.*—Stand a prism on a drawing-board as in § 42 and draw a series of lines to represent rays incident on one face as in Fig. 52. By sticking pins into these lines $P_1Q_1$, $P_2Q_2$, $P_3Q_3$, and by setting up other pins $R_1S_1$, $R_2S_2$ and $R_3S_3$ so that each set of four pins (PQRS) appears in line, these rays can be traced through the prism and a diagram like Fig. 52 obtained.

The angle of deviation and the angles of incidence and emergence can then be measured with a protractor; results such as are given in the following table will be obtained:—

| | | | | | |
|---|---|---|---|---|---|
| Angle of incidence $= i$ . | 30° | 40° | 45° | 50° | 55° |
| Angle of emergence $= r'$ . | 77° | $58\frac{1}{2}$° | $52\frac{1}{4}$° | $47\frac{1}{4}$° | $42\frac{3}{4}$° |
| Angle of deviation $= D$ . | 47° | $38\frac{1}{2}$° | $37\frac{1}{4}$° | $37\frac{1}{4}$° | $37\frac{3}{4}$° |

Two facts are clear from this table.

(i.) As the angle of incidence increases the angle of emergence

decreases. These two angles are equal when $i = r' > 45°$ and $< 50°$. By drawing a graph (Fig. 53, Graph I.) we find that they are exactly equal when $i = r' = 48\frac{3}{4}°$.

(ii.) The deviation is greatest when $i$ and $r'$ are most unequal, and least when $i$ and $r'$ lie between 45° and 50°. From Graph II. (Fig. 53) we see that D is least when $i = r' = 48\frac{3}{4}°$.

We can thus apply a practical test to the theorem of § 43 that *the*

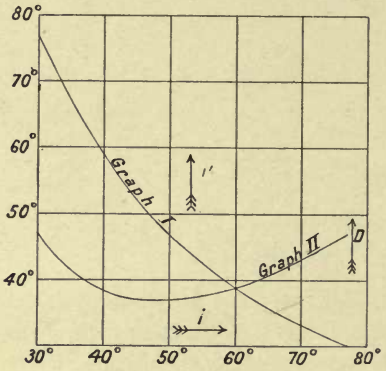



Fig. 53.

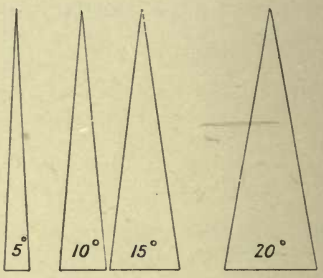



Fig. 54.

*deviation is least when the angles of incidence and emergence are equal, i.e., when the ray passes through symmetrically.*

It was shown in § 42 that

$$i + r' = D + A$$

$$\text{and } i' + r = A.$$

Now when the ray passes through symmetrically

$$i = r'$$

$$\text{and } i' = r$$

$$\therefore i = \frac{D + A}{2} \quad \text{and } r = \frac{A}{2}.$$

The angle of the prism can be found experimentally by tracing rays reflected from its faces just as the angle between two mirrors was found in § 18.

Suppose this is 60°.

$$\text{Then we have } i = \frac{D + A}{2} = \frac{37\frac{1}{4}° + 60°}{2} = 48° 45'$$

$$\text{and } r = \frac{A}{2} = \frac{60°}{2} = 30°$$

$$\therefore \mu = \frac{\sin 48° 45'}{\sin 30°} = \frac{\cdot 7519}{\cdot 5000} = 1\cdot 504.$$

In practice this is one of the most accurate ways of determining $\mu$.

§ 45. *Refraction at Curved Surfaces. Lenses.*—Suppose that we have a number of prisms all of the same glass, but having different vertical angles. Such a set is shown in Fig. 54. The deviation produced in a ray passing through any one of these depends partly on its angle of incidence, but mainly on the angle of the prism.

The following table shows the deviations produced by such a set of prisms made of a kind of glass for which $\mu = 1{\cdot}5$ :—

| Angle of Prism . | 0° | 10° | 20° | 30° | 40° | 50° | 60° |
|---|---|---|---|---|---|---|---|
| Angle of incidence— | | | | | | | |
| 10° . . . | 0° | 5° 15′ | 10° 30′ | 16° 40′ | 35° 50′ | — | — |
| 20° . . . | 0° | 5° 15′ | 9° 40′ | 15° 45′ | 22° 40′ | 34° 0′ | — |
| 30° . . . | 0° | 6° 50′ | 10° 45′ | 15° 50′ | 21° 40′ | 29° 35′ | 46° 50′ |
| 40° . . . | 0° | 6° 40′ | 12° 0′ | 17° 0′ | 22° 20′ | 28° 45′ | 38° 35′ |
| 50° . . . | 0° | 8° 0′ | 13° 55′ | 19° 0′ | 24° 5′ | 29° 45′ | 37° 20′ |

It is therefore clear that by cutting slices from such a set of prisms and fitting them together as in Figs. 55A, 55B, and 55C, we could cause light starting from such a point as O to be so deviated as mainly to go to I, or to appear to have come from I.

Whether I lies beyond the bundle of prism sections or on the same side as O will depend both on the deviating power of the prisms and on the angles of incidence of the rays, *i.e.*, on the position of O.

In any case the rays from O are originally divergent; the effect of the bundle of prism sections is to make the rays less divergent, and the extent to which the divergence is decreased may result in (i.) less divergence (Fig. 55A); (ii.) parallelism (Fig. 55B); or (iii.) convergence (Fig. 55C).

There is another way in which the sections may be grouped; this is shown in Fig. 56. In this case the beam is always rendered more divergent, and therefore I must lie between O and the bundle of prisms.

In practice an optical contrivance such as the bundle of prisms would not answer well, but a similar effect is produced by grinding the surfaces of a sheet of glass so that they are portions of spheres. This may be done in various ways as is shown in section in Fig. 57, A, B, C, D, E and F.

§ 46.—If these are examined it will be seen that A, B, and C are equivalent to the first arrangement of prism sections (Fig. 55) and

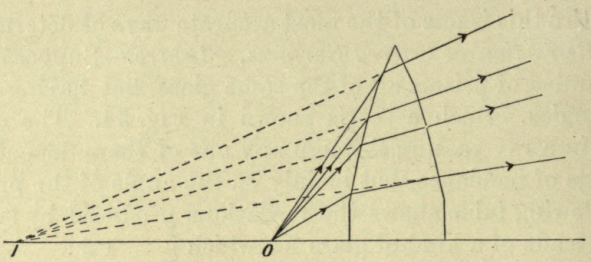



Fig. 55A.

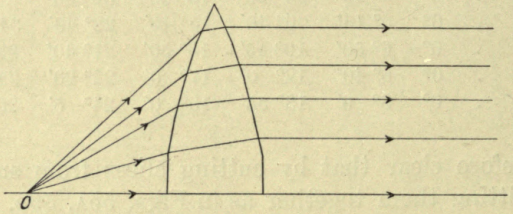



Fig. 55B.

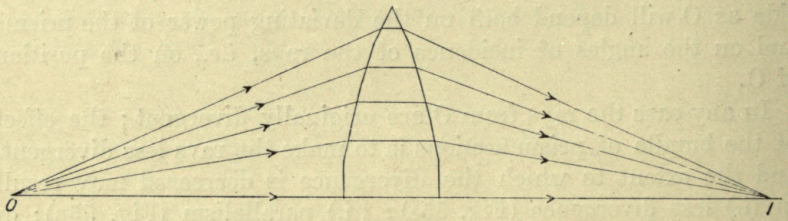



Fig 55C.

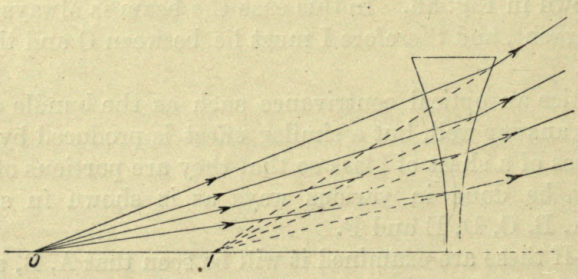



Fig. 56.

share with that arrangement the peculiarity of being thickest in the middle. *A lens which is thicker at the centre than near the edges is called a* **Convex Lens.**

D, E, F (Fig. 57) share with the second arrangement of prisms the peculiarity of being thinnest in the middle. *A lens which is thinner at the centre than near the edges is called a* **Concave Lens.**

By comparison with the bundle of prism sections we can see that when a bundle of divergent rays falls on a convex lens the rays will be so bent as to be less divergent and so may (i.) diverge as though from an image further from the lens than the object; (ii.) become parallel; or (iii.) converge to an image beyond the lens. Divergent rays are always rendered more divergent by a concave lens, and so must always appear to come from an image between the lens and object.

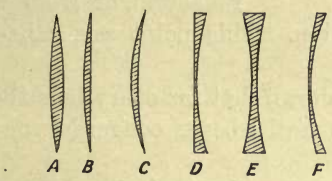



Fig. 57.

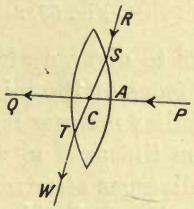



Fig. 58.

§ 47. *Thin Lenses. Centre of Lens.*—In the future we shall deal only with lenses which are so thin that their thickness may be neglected in comparison with the distances from the lens of the objects used and images formed.

On going back to our lens of prism sections we see that the central section in every case is of a prism whose angle is 0°. That is to say, the central section has parallel faces. A ray going through a piece of glass whose faces are parallel is not bent or deviated, but is shifted slightly to one side (§§ 33 and 35). If the piece of glass is thin, this shifting is negligible. *We may therefore say that rays passing through the centre of a lens are not deviated.*

If the lens is of some thickness, as is the one shown in Fig. 58, a ray such as PAQ will not be deviated. It is also possible that a ray such as RSTW may enter and leave at points S and T, far removed from A, but yet so placed that the surfaces of the lens at S and T are parallel. In this case the ray RSTW is not bent, but only shifted.

By investigating such rays as RSTW and PAQ it is possible to find a point C in the lens such that all rays which pass through C are undeviated. This point C lies on the line joining the centres of curvature of the faces of the lens.

*The line joining the centres of curvature of the faces of a lens is called the Axis of the Lens.*

*The point on the axis through which those rays pass which are not deviated by passing through the lens is called the Centre of the Lens.*

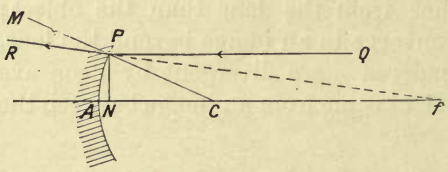



Fig. 59A.

§ 48. *Principal Focus.* — In considering the reflection of light at a curved surface it was shown that there is a point to which a parallel beam converges after reflection, or from which it appears to diverge after reflection. This point was called the principal focus of the mirror.

In a similar manner it may be shown that for every lens there is a point on the axis to which a parallel beam converges, or from which it diverges after refraction.

In order to find this point mathematically it is necessary to consider the refraction at each surface separately.

In Fig. 59 are shown the two possible cases. In Fig. 59A the lens surface is concave. C is the centre of curvature and CA is the axis. By the rule of signs CA = + $r$ where $r$ is the radius of curvature.

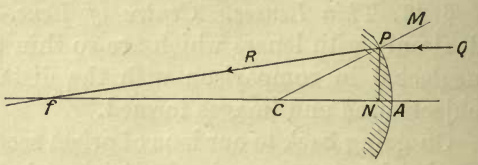



Fig. 59B.

The ray QP is parallel to CA, and by the ordinary rule of refraction must cross the normal at P and be bent nearer to it, so that

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \text{QPC}}{\sin \text{RPM}} = \frac{\sin \text{PCA}}{\sin f\text{PC}}.$$

The ray after refraction will therefore appear to come from $f$. By the rule of signs we may put A$f$ = + $f$.

In Fig. 59B the surface is convex and the ray is bent towards

the axis so as to cut it in $f$. By the rule of signs in this case $CA = -r$ and $Af = -f$.

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \text{QPM}}{\sin f\text{PC}} = \frac{\sin \text{PCA}}{\sin f\text{PC}}.$$

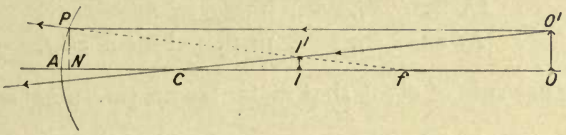



Fig. 60A.

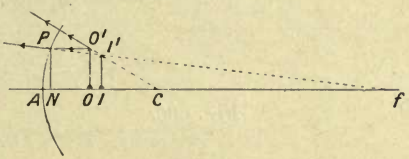



Fig. 60B.

Now $\sin$ PCA $= \sin$ (180° − PCA) $= \sin$ P$Cf$, and by the formula of Appendix II., § 5,

$$\frac{\sin \text{P}Cf}{\sin f\text{PC}} = \frac{\text{P}f}{f\text{C}}.$$

We have therefore

$$\text{P}f = \mu \cdot f\text{C};$$

or approximately, where P is near to A,

$$Af = \mu \cdot f\text{C};$$

or $f = \mu(f - r)$ in Fig. 59A, and $-f = \mu(-f - (-r))$ in Fig. 59B.

These two formulæ both give the same result, viz.:—

$$f(\mu - 1) = \mu r,$$

$$\text{or } f = \frac{\mu r}{\mu - 1}.$$

§ 49.—In Fig. 60 are shown some cases in which the incident rays are not parallel, but diverge from an object OO′. A ray starting from O′ and going through C the centre of curvature will not be deviated since it will be normal to the refracting surface.

A ray O′P parallel to CA will either pass through $f$ or will appear to have come from $f$. The image of O′ will therefore be at I′ where O′C and P$f$ (or these lines produced) meet.

By carrying out a similar construction for various points in OO′

the images of these points will be found to lie along a line II′ perpendicular to OC produced.

Making use of the following symbols :—

$$\begin{aligned} \text{OA} &= \text{distance of object} = u, \\ \text{IA} &= \quad \text{,,} \quad \text{,, image} = v, \\ \text{A}f &= \quad \text{,,} \quad \text{,, principal focus} = f, \end{aligned}$$

and the usual rule of signs that distances on the same sides of the

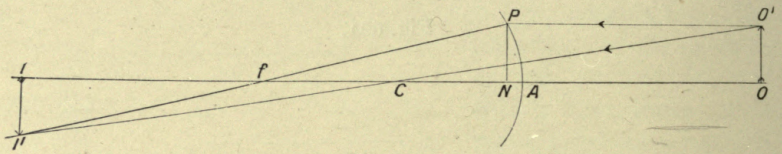



Fig. 60c.

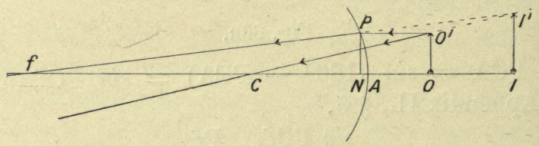



Fig. 60d.

surface as the object are $+^{ve}$ while those on the opposite side are $-^{ve}$, we have from the similarity of the triangle OO′C and II′C

$$\frac{\text{OO}'}{\text{II}'} = \frac{\text{OC}}{\text{IC}},$$

and from the similarity of the triangles PN$f$ and I′I$f$

$$\frac{\text{PN}}{\text{II}'} = \frac{\text{N}f}{\text{I}f}.$$

But PN = OO′,

$$\therefore \frac{\text{OC}}{\text{IC}} = \frac{\text{N}f,}{\text{I}f}$$

$$\text{or } \frac{u-r}{v-r} = \frac{f}{f-v} \text{ (Figs. 60a and 60b)},$$

$$\frac{u+(-r)}{(-v-(-r))} = \frac{-f}{(-v-(-f))} \text{ (Fig. 60c)},$$

$$\frac{u+(-r)}{v+(-r)} = \frac{-f}{v+(-f)} \text{ (Fig. 60d)}.$$

All these reduce to

$$(u - r)(f - v) = f(v - r),$$
$$\text{or } uf - uv - rf + rv = fv - fr,$$
$$\text{or } uf - uv + rv - fv = 0;$$

but $f = \frac{\mu r}{\mu - 1}$,

$$\therefore \frac{\mu ur}{\mu - 1} - uv + rv - \frac{\mu vr}{u - 1} = 0,$$

or, dividing through by $uvr$,

$$\frac{\mu}{\mu - 1}\frac{1}{v} - \frac{1}{r} + \frac{1}{u} - \frac{\mu}{\mu - 1}\frac{1}{u} = 0,$$

which may be more simply written

$$\frac{\mu}{v} - \frac{\mu - 1}{r} - \frac{1}{u} = 0,$$

$$\text{or } \frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}. \quad \text{(Equation i.)}$$

§ 50.—Now suppose that the light meets a second surface whose radius of curvature is $s$. The image formed by the first refraction will serve as object for the second refraction. The refractive index from glass to air will be $\frac{1}{\mu}$. Hence, if $V$ is the distance of the final image,

$$\frac{\frac{1}{\mu}}{V} - \frac{1}{v} = \frac{\frac{1}{\mu} - 1}{s},$$

$$\text{or } \frac{1}{V} - \frac{\mu}{v} = \frac{1 - \mu}{s}. \quad \text{(Equation ii.)}$$

If equations (i.) and (ii.) be added together we get

$$\frac{1}{V} - \frac{1}{u} = \frac{\mu - 1}{r} - \frac{\mu - 1}{s} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

*If $u$ is made infinitely great so that the incident beam is parallel, then the light will be brought to a focus at the principal focus, the distance of which from the lens is called the focal length of the lens.*

We have in this case

$$\frac{1}{u} = \frac{1}{\infty} = 0, \text{ and } V = F,$$

$$\therefore \frac{1}{F} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

and the general formula may be written

$$\frac{1}{\mathbf{V}} - \frac{1}{\mathbf{U}} = \frac{1}{\mathbf{F}},$$

§ 51. *Practical Proof of the Formula for a Convex Lens.*—Set up a convex lens on a stand between yourself and a distant object.

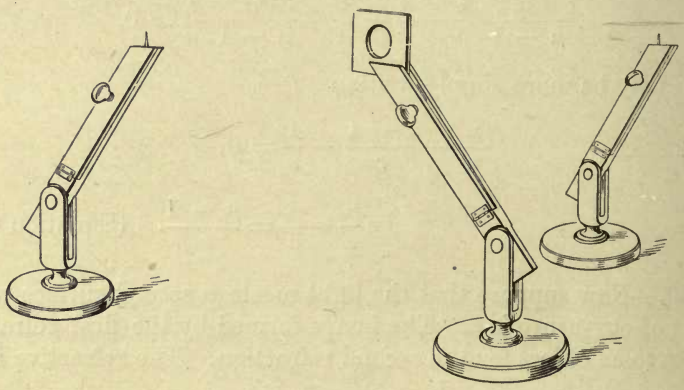


Fig. 61.

When your eye is at some distance from the lens and in line with the lens and object a small inverted image can be seen. Place a pin so that it coincides with this image (*see* § 26). Measure the distance of the pin from the lens. Suppose this distance is 16 cms. Then, since the image is on the opposite side of the lens to the object, $F = -16$ cms.

Set up the lens on a firm stand and a pin on another stand so that the pin is at about the same height as the centre of the lens and about 50—100 cms. away from it. On looking as before, a real inverted image of the pin can be seen. Place a second pin so as to coincide with the image. Measure the distances of the pins from the lens. Suppose that

Distance of first pin (object) = 50 cms.
„ second pin (image) = 23·5 cms.

Then $U = 50$ cms.

$V = -23{\cdot}5$ cms. since image is on opposite side to object.

And by the formula

$$\frac{1}{F} = \frac{1}{V} - \frac{1}{U} \text{ or } F = \frac{UV}{U - V},$$

$$\therefore F = \frac{50\ (-\ 23{\cdot}5)}{50\ -\ (-\ 23{\cdot}5)} = -\ \frac{50 \times 23{\cdot}5}{73{\cdot}5}$$

$$= -\ 15{\cdot}99 \text{ cms.}$$

By altering the distance of the object-pin and resetting the image-pin, other values of $V$ and $U$ can be found and $F$ again calculated. In every case one should find $F$ to be the same (viz.:—about $-16$ cms.).

Verification of the formula for a concave lens is more difficult, since a real image cannot be formed and it is not possible to set a pin so as to coincide with a virtual image in the ordinary way; but by cutting a hole in the lens this is possible. The image-pin is viewed through the hole and the object-pin through the lens. The appearance presented is shown in Fig. 62. In this case, since both $V$ and $U$ are positive and since $U > V$, the values obtained for $F$ will also be positive.

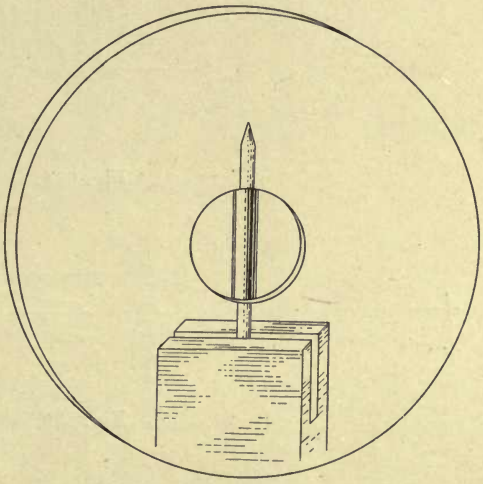


Fig. 62.

§ 52. *Graphical Construction for Lenses. Magnification.*—Suppose that we have an object OO′ placed in front of a lens (Fig. 63). Then we know that a ray O′P parallel to the axis OC will, after refraction, pass through F, the principal focus, or appear to have come from F.

A ray O′C passing through the centre of the lens (§ 47) will not be deviated.

Hence the image of O′ will lie at I′ where FP and CO′ (or these lines produced) meet.

We are thus able to find the position of I′, the image of O′, by a purely geometrical construction.

If other points are taken in OO′ and the positions of their images constructed, these will be found to lie in II′ where II′ is perpendicular to the axis of the lens, provided always that OO′ is small compared with its distance from the lens.

In using this construction for solving problems connected with lenses care must be used to draw the lenses *very* thin; those in Fig. 63 are much too thick for accurate work.

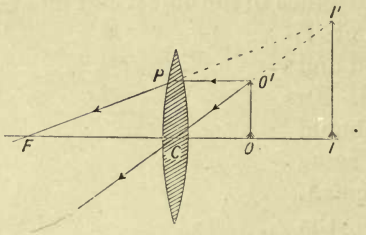



Fig. 63A.

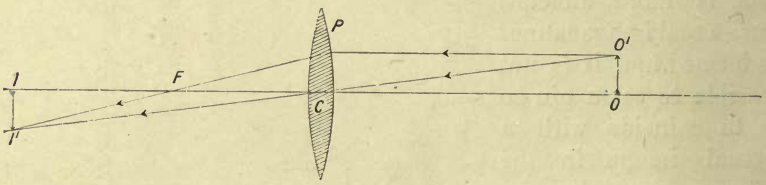



Fig. 63B.

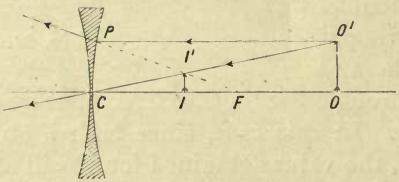



Fig. 63C.

§ 53.—In all the above figures the triangles OO′C and II′C are similar.

$$\therefore \frac{II'}{OO'} = \frac{IC}{OC} = \frac{V}{U},$$

or the ratio of the lengths of the image and object = the ratio of their distances from the lens.

In § 27 it was shown that exactly the same rule holds for mirrors. In the case of both lenses and mirrors, therefore, linear magnification $= \frac{V}{U}$.

§ 54. *Another, but less rigid Proof of the Formula for Lenses.*— From the figures of § 52 (Figs. 63A, 63B and 63C) it is clear that the triangles I'IF and PCF are similar.

$$\therefore \frac{\text{II}'}{\text{CP}} = \frac{\text{IF}}{\text{CF}},$$

but CP = OO', and $\frac{\text{II}'}{\text{OO}'} = \frac{V}{U}$ (§ 53),

$$\therefore \frac{\text{IF}}{\text{CF}} = \frac{V}{U},$$

or $\frac{V}{U} = \frac{V + (-F)}{-F}$ (Fig. 63A), $\frac{-V}{U} = \frac{-V-(-F)}{-F}$ (Fig. 63B),

$$\frac{V}{U} = \frac{F - V}{F} \text{ (Fig. 63C).}$$

All these reduce to

$$VF = UF - UV,$$

$$\text{or } \frac{1}{U} = \frac{1}{V} - \frac{1}{F}, \textit{ i.e.}, \frac{1}{V} - \frac{1}{U} = \frac{1}{F}$$

This proof is not so rigid as that given in §§ 48, 49 and 50, since it assumes, but does not prove, that there is a point F to which parallel rays go after refraction.

§ 55. *Kinds of Images formed by Various Kinds of Lenses.*—In § 45 it was pointed out that a convex lens may form a real or a virtual image, while a concave lens can only give a virtual image. This may also be shown from the formulæ of §§ 50 and 54.

Consider the lenses of Fig. 57, and let us throughout suppose that an object is on the right-hand side.

The radii of curvatures of the right-hand faces will be designated $r$; those of the left-hand faces, $s$.

Consider first the formula

$$\frac{1}{F} = \overline{\mu - 1}\left(\frac{1}{r} - \frac{1}{s}\right)$$

$$\text{or } F = \frac{rs}{(\mu - 1)(s - r)}$$

In lens A, $r$ is $-^{ve} = -r'$
$s$ is $+^{ve} = +s'$

$\therefore \dfrac{-r's'}{(\mu-1)(s'+r')}$, *i.e.* $F$, has a $-^{ve}$ value.

In lens B, $r$ is $\infty$
$s$ is $+^{ve}$

$\therefore \dfrac{-s}{\mu-1}$, *i.e.* $F$, has a $-^{ve}$ value.

In lens C, $r$ is $+^{ve}$
$s$ is $+^{ve}$ but $r > s$

$\therefore \dfrac{rs}{(\mu-1)(s-r)}$ is $-^{ve}$, *i.e.* $F$, has a $-^{ve}$ value.

In lens D, $r$ is $+^{ve}$
$s$ is $\infty$

$\therefore \dfrac{r}{\mu-1}$, *i.e.* $F$, has a $+^{ve}$ value.

In lens E, $r$ is $+^{ve}$
$s$ is $-^{ve} = -s'$

$\therefore F = \dfrac{r(-s')}{(\mu-1)(-s'-r)} = \dfrac{rs'}{(\mu-1)(s'+r)}$, *i.e.* $F$, has a $+^{ve}$ value.

In lens F, $r$ is $+^{ve}$
$s$ is $+^{ve}$ but $r < s$

$\therefore \dfrac{rs}{(\mu-1)(s-r)}$ is $+^{ve}$, *i.e.* $F$, has a $+^{ve}$ value.

As these are the only possible cases it is clear that we may state the case generally by saying

$F$ is $-^{ve}$ for all convex lenses,
$F$ is $+^{ve}$ for all concave lenses.

§ 56.—Moreover, the formula of § 50 may be written

$$\frac{1}{V} = \frac{1}{U} + \frac{1}{F},$$

$$\text{or } V = \frac{UF}{F+U} = U\frac{F}{U+F}.$$

Hence,

(i.) For a concave lens,

Since $F$ is $+^{ve}$, $V$ is always $+^{ve}$ and less than $U$.

(ii.) For a convex lens

$F$ is $-^{ve}$,

$\therefore$ $V$ is $-^{ve}$ when $U$ is numerically greater than $F$, and $V > U$ when $F > U - F$, *i.e.*, when $U < 2F$.

$V$ is $+^{ve}$ when $U$ is numerically less than $F$, and $V$ is always greater than $U$.

§ 57. *Upright and Inverted Images.*—Suppose that BCA (Fig. 64) represents the position of any sort of lens, and that OO′ is an object placed near the lens. From former sections we know that the image may have any one of the three positions of II′ shown in the figure. In all cases I′ will lie on O′C or on O′C produced where C is the centre of the lens. It is at once apparent from the figure

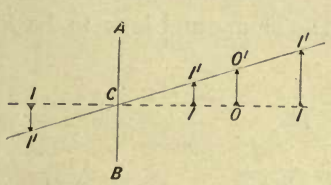



Fig. 64.

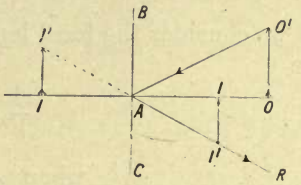



Fig. 65.

that when II′ is on the same side of the lens as OO′, then II′ is the same way up as OO′; when II′ is on the opposite side of the lens, then II′ is upside down as regards OO′. Hence the general rule—*Real images formed by lenses are inverted; virtual images are erect.* In making experiments with lenses, this rule is often of great use as a practical guide to the position and nature of the image seen.

As a matter of fact the *same rule holds for the images formed by reflection in a mirror*, as is readily proved from Fig. 65. Let CAB be the position of a mirror (either concave or convex), and let OO′ be an object. The ray O′A is reflected along AR according to the laws of reflection. The image of O′ must therefore lie in AR or in RA produced. Hence, the image of OO′ can have either of the positions of II′ shown. If it is behind the mirror it is erect; when the image is real (in front) it is inverted. Hence the above rule.

§ 58. *Combination of Lenses.*—If two thin lenses are placed very close together so that they may be considered as a single lens there

is a simple formula connecting the focal length of the compound lens, and the focal lengths of the component lenses.

Let $U$ be the distance of the object; light falling on the first lens is refracted so as to give an image at a distance $V_1$ such that

$$\frac{1}{V_1} - \frac{1}{U} = \frac{1}{F_1}$$

where $F_1$ is the focal length of the lens.

Light now comes to the second lens as though from $V_1$; hence,

$$\frac{1}{V} - \frac{1}{V_1} = \frac{1}{F_2}$$

where $F_2$ is the focal length of the second lens.

On adding these two equations together $V_1$ is eliminated, and we get

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{F_1} + \frac{1}{F_2}.$$

If we consider the focal length of the compound lens to be $F$ we have

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{F},$$

$$\text{whence } \frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}.$$

§ 59. *Experimental Determination of Focal Length of a Concave Lens.*—The formula of the last paragraph leads to a simple method for finding the focal length of a concave lens by combining it with a convex lens of known focal length. Using the method of § 51 we can find the focal length of a single convex lens. Suppose that this is — 16 cms. On clamping this to our given concave lens, we find by the same method that the focal length of the compound lens is — 30 cms.

$$\therefore \frac{1}{-30} = \frac{1}{-16} + \frac{1}{F_2},$$

$$\text{or } F_2 = \frac{1}{\frac{1}{16} - \frac{1}{30}} = \frac{480}{14} = +34{\cdot}3 \text{ cms.}$$

Notice that in order to carry out the experiment successfully the compound lens must be equivalent to a simple *convex* lens, *i.e.*, $F$ must be $-^{ve}$; if this is so $F_2$ is numerically greater than $F_1$.

*Examples.*

1. What is meant by the Index of Refraction of a medium?

Why does a stick partly immersed in water appear bent? (Army 02.)

2. A plane mirror lies horizontally. The image of a point lying above the mirror is viewed by an eye looking at an angle of 45° with the mirror, the height of the eye above the mirror being twice the height of the point. Draw a diagram to indicate the path of two rays proceeding from the point to the eye.

Explain whether the eye would still see the image of the point in the same direction if the mirror lay at the bottom of a tank, and if water were poured into the tank (*a*) so as to just cover the point, and (*b*) so as to reach up to the eye. (Matric. 08.)

3. State carefully the laws of refraction of light, and explain how you can verify them experimentally by means of a rectangular block of glass, some pins, paper, a scale, set square, and a pair of compasses. (Matric. 08.)

4. State the laws of refraction of light.

A ray of light is incident on the surface of water at an angle of 45°. Find by a diagram the direction of the refracted ray. The index of refraction of water is 1·33. (Camb. Loc. Jun. 06.)

5. A plate of glass 1 in. thick and of refractive index $\frac{3}{2}$ is placed over a small object which is viewed by an eye above the plate and in a nearly perpendicular direction; find approximately, by drawing, how much nearer the object appears. (Oxf. Loc. Sen. 06.)

6. An object at the bottom of a tank of water is observed from above. Does its apparent position depend on the position of the observer? Draw a diagram to illustrate your answer. (Camb. Loc. Sen. 07.)

7. A microscope was focussed upon a mark on the surface of a thick glass mirror. It was then moved toward the mirror till the image of the mark was in focus. The distance the microscope was moved was 1·06 cm. Find the thickness of the glass, its refractive index being 1·50. (Camb. Spec. 07.)

8. Explain why a ray cannot always be refracted from one medium to another.

By a graphic construction determine the critical angle of refraction from glass to air, the refractive index of glass being 1·5. (Oxf. Prelim. and C. (3) 04.)

9. Light is incident on one face of a rectangular block of glass whose refractive index is 1·5. Show that no light can emerge from the block except at the opposite face or the face on which the light was incident. (Camb. Loc. Sen. 07.)

10. What are the conditions under which total reflexion may occur?

Describe an experimental method of determining the refractive index of a liquid by measurement of its critical angle. (Oxf. Loc. Sen. 05.)

11. Describe experiments to show that light travels in straight lines. What do you understand by a ray of light? Draw a diagram to show the course of a ray of light through a prism. (Camb. M.B. 06.)

12. Light falls at grazing incidence on a prism of angle 30° and of refractive index $\frac{3}{2}$; construct the emergent ray, and measure approximately its inclination to the normal.

What rays incident at one face will not emerge at the other without internal reflexions? (Oxf. Loc. Sen. 07.)

13. What are the laws of refraction of light? When an observer looks at distant objects through an ordinary window pane, the objects are considerably distorted if the observer is at some distance from the window, but if the observer is close to the window the distortion is hardly noticeable. Explain these facts. (Camb. Loc. Jun. 07.)

14. Show by means of a diagram the course of a small parallel pencil of light through a prism, and find the angle of minimum deviation when the refractive index is $\mu$ and the angle of the prism $\epsilon$. (Camb. Spec. 03.)

15. Explain the formation of images by means of a small aperture. What advantages and disadvantages would such an aperture have as compared with a lens? (Oxf. Loc. Sen. 07.)

16. What is meant by (1) a focus, (2) a principal focus, of a lens? Describe the conditions under which (1) a real, (2) a virtual image can be formed by a convex lens. (Camb. M.B. 05.)

17. Show that the least distance between an object and its image formed by a convex lens is equal to four times the focal length of the lens. (Army 01.)

18. Compare the images formed by a concave mirror and a convex lens. What difference may be observed between them. (Camb. Loc. Sen.)

19. Define the index of refraction of a transparent medium.

A small object is placed at a distance of 20 cms. from a thin double convex lens and on the axis; the focal length of the lens is 12 cms.; where is the image formed, and how much larger is it than the object? (Oxf. Loc. Sen. 08.)

20. Draw, carefully, the path through a sphere of water of a pencil of rays proceeding from a luminous point, the distance of the point from the centre of the sphere being twice the diameter of the sphere. The refractive index of water is 1·33. (Army 01.)

21. A converging lens has one of its faces plane, and the plane face is silvered. The lens is held in the path of light diverging from a small hole in a screen, the curved surface being towards the hole. When the lens is at a certain distance from the screen, a sharp image of the hole is thrown on it. Explain, by means of a drawing, the path of the rays in their course from the hole to the image. What is the distance of the lens from the screen? Would the effect be the same if the experiment were repeated with the curved face of the lens silvered and the plane face turned towards the hole? Give reasons. (Matric. 08.)

22. A convex lens of 2 ins. focal length is held 1 in. from the eye by a person with distance of distinct vision of 9 ins. so as to look at a small object. Where must the small object be placed? Illustrate your answer by a figure. (Lond. Inter. Sci. 02.)

23. Show that if a horizontal concave mirror is filled with a liquid its apparent radius of curvature is diminished in the ratio of the refractive index of the liquid. (Lond. Inter. Sci. 07.)

24. When a lens is held close to a printed page and moved in any direction parallel to the page, the image of the letters seen through the lens moves in the same direction if the lens is concave, and in the opposite direction if the lens is convex. With the aid of diagrams explain these observations. (Camb. Spec. 07.)

25. State the properties of a focus and of the centre of a lens, and utilize them to determine graphically the position of the image given by a convex lens of a

small object placed on its axis at a less distance from it than its focal length.

How far in front of a concave lens, of focal length 3 ins., is an object when its image is virtual and of one-third its size? (Oxf. Prelim. and C. (3) 08.)

26. How would you determine the focal length of a convex lens?

Prove the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. (Camb. Loc. Sen. 05.)

27. A pin, 5 mm. long, is placed at a distance of 2 cms. from a convex lens, and an image is formed, 1 cm. long, on the other side of the lens. Find, *by drawing an enlarged diagram to scale*, the focal length of the lens. Explain your construction, and state your scale. (Oxf. Loc. Jun. 04.)

28. A candle flame is placed on the principal axis of a convex lens of 24 ins. focus, and is 6 ft. from the lens. State fully the character, size, and position of the image which will be formed. Describe what would be seen by an observer whose eye is on the principal axis, 4 ft. from the lens, and on the side opposite to the candle. (Oxf. Prelim. and C. (3) 03.)

29. It is required to project a 3 in. × 3 in. slide on to a large screen 40 ft. away. What focal length should the projecting lens have and how far would it be from the slide, when an image measuring 6 ft. × 6 ft. is thrown on the screen? (Army 05.)

30. Describe a practical method of finding the focal length of a convex mirror.

An object is placed 8 cms. to the right of a convex lens of focal length 4 cms. After refraction the rays strike a convex mirror of 6 cms. radius, placed 10 cms. to the left of the lens. By tracing the rays from the object to scale, find the position of the image formed after reflexion in this mirror. State your construction. (Oxf. Loc. Sen. 05.)

31. How would you arrange a convex lens to give you (1) a magnified real image, (2) a magnified virtual image? How would you distinguish between a real and a virtual image? (Camb. Loc. Jun. 05.)

32. Draw a curve showing in the case of a convex lens the connection between the distance of the object from one principal focus and the distance of the image from the other. (Lond. Inter. Sci. 03.)

33. Trace the changes which occur in the size and position of the image formed by a doubly convex lens as the object moves from an infinite distance up to the lens. (Army 00.)

34. Explain how the image of an object produced by refraction through a lens can be graphically determined from a knowledge of the focal length of the lens.

If the lens is convex, where must the object be placed for its image to be erect? (Oxf. Prelim. and C. (3) 05.)

35. When a candle is placed at a certain distance in front of a convex lens, and the lens is viewed from the same side as the candle, two erect images of the candle are seen. Explain their formation. (Camb. Loc. Sen.)

36. When a lamp and scale are used in measuring the deflexion of a galvanometer mirror, light passes through a slit across which a vertical wire extends, and then, passing through a convex lens, falls upon a plane mirror; after reflexion, the light passes above the lens and falls on the scale, which is over the slit. Explain how the lens must be placed in order that a clear inverted image of the wire may be formed on the scale, and show that the focal length

of the lens must be less than half the distance of the scale from the mirror. (Camb. Spec. 07.)

37. Describe a practical method of determining the focal length of a concave lens.

A concave lens of focal length 8·4 cms. is placed in front of an object at a distance 11·2 cms. from it. Make a careful drawing to scale, showing the position of the image and the path of the rays from the object through and beyond the lens. Is the image real or virtual, erect or inverted? (Oxf. Loc. Sen. 04.)

## CHAPTER IV

### OPTICAL INSTRUMENTS

§ 60. *The Camera Obscura.*—The camera obscura is a simple combination of a mirror and a convex lens forming a real image. In Fig. 66 are shown the various portions of the instrument in section.

A lens L forms an image of OO′. This image would be as shown

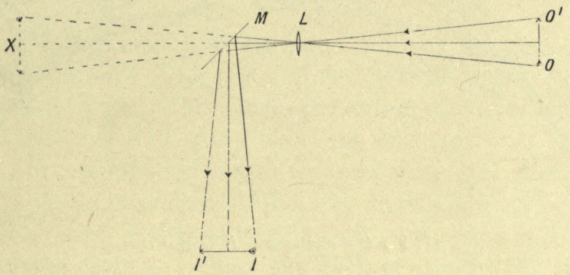



Fig. 66.

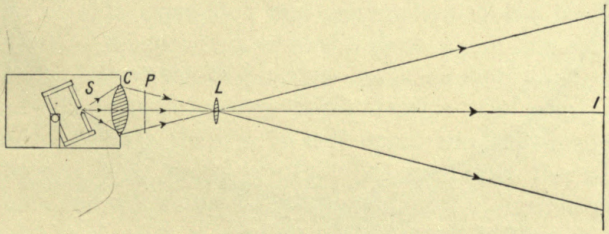



Fig. 67.

by the dotted lines at X were the rays not diverted by reflection at the mirror M so that the image is actually at II′, where a screen is placed to receive it.

§ 61. *The Optical Lantern.*—This is shown in section in Fig. 67. The source of light S sends out rays in all directions. Some of these rays fall on the powerful convex lens, or "condenser," C, and are thereby rendered convergent towards a point somewhere in front of C.

The convergent beam from C then passes through the transparent slide or picture P, and so to the lens L, which is convex and so placed as to cast a real inverted image of P on the screen I.

§ 62. *The Human Eye.*—The human eye consists of a convex lens and a screen. Images of distant objects are formed on the screen by the lens, and there excite nerves communicating with the brain. A section of the eye is shown in Fig. 68.

The outer horny coat, or *Sclerotica*, has a transparent and slightly more curved portion in front called the *Cornea*. Light entering the eye is refracted at the cornea and again by the gelatinous *Lens*,

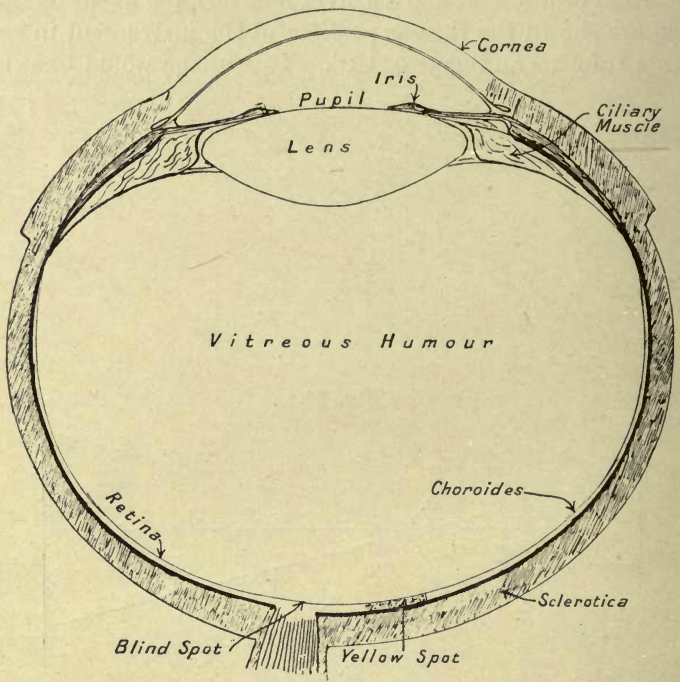



Fig. 68.

and so forms an image on the nerve-covered coating at the back of the eye called the *Retina*. But we know that the position of the image depends on the distance of the object ($U$) and also on the focal length of the lens ($F$). Now in the eye the distance from the lens to the retina is fixed; in order, therefore, that the images of objects at various distances may be clearly formed on the retina, arrangements are provided for altering the focal length of the lens. This is done by means of the *Ciliary Muscle* which surrounds the lens. When this muscle contracts, the lens is squeezed from the sides,

and therefore expands from front to back, and becomes more highly curved. Now the more highly curved the faces of a lens, the shorter is its focal length, and the smaller is $U$ for a given value of $V$. When the ciliary muscle slackens, the surfaces of the lens become flatter, and objects at a greater distance can be clearly focussed.

There is clearly a limit to this squeezing and flattening process. In the normal eye the lens can be so flattened that objects at an infinite distance are brought to a clear focus on the retina; there remains, therefore, only one limit to distinct vision—when the lens is squeezed to its utmost to accommodate for extreme nearness of the object. The *least distance of distinct vision* for a normal eye is about 6 ins.

The retina is very sensitive, and is blinded by the glare of a very brilliant illumination. The *Iris Diaphragm* is therefore provided to cut off excess of light. This forms the coloured band between the "white" of the eye and the *Pupil*. If the eyes of a person are observed (i.) in a bright light, and (ii.) in a dim light, it will be seen that when the light is bright the iris diaphragm covers a large portion of the pupil so that only a few rays can pass through from the centre of the cornea. When the light is dim the iris diaphragm recedes, allowing light to enter through every portion of the lens. The effects of this expansion and contraction of the iris must be familiar to all. When one goes into a dark room, one can at first see very little, but as the diaphragm opens, enough light enters the eye to enable objects to be seen fairly clearly. On going out into bright sunshine again the iris is still open, and the image on the retina is so bright as to be painful; but gradually the iris draws over the lens and allows only the right amount of light to pass.

When the back part of the eye is more closely examined it is found that inside the sclerotica is a second thinner protective coat called the *Choroides*. On this is spread a system of "rods and cones" which are really nerve-ends forming the true *Retina*. This system of fibres is seen more highly magnified in Fig. 69. When light falls on the tips of these a nerve impulse is forwarded to the brain along the nerves at the back of the retina and up the bundle of nerves which leave the eye at the *Blind Spot* (Fig. 68). At this spot the nerves from all over the retina leave the eye and there are no projecting nerve-ends in which light may cause a sensation.

Close to the blind spot is the *Yellow Spot;* this is a spot where for some reason or other the nerves are more sensitive than

anywhere else on the retina. Consequently one instinctively so uses his eyes that the image of the centre of the view comes on to the yellow spot. It is clear that the image of some neighbouring object will fall on the blind spot, and this is easily shown by the numerous optical illusions in which one object is invisible when attention is concentrated on another.

§ 63. *Diseases of the Eye: Myopia, Hypermetropia, Astigmatism.* —In the last section it was pointed out that in order to see distant objects the ciliary muscle must be slackened until the lens has become sufficiently flat. In some eyes the lens retains a certain curvature when the muscle is slackened; this may either be due to too constant squeezing by the muscle, *i.e.*, too much looking at near objects, or may be a natural defect. In either case an eye whose lens is too highly curved cannot clearly focus distant objects. Such an eye is said to be *short-sighted* or *myopic*. Since the lens is too convex its defect can be compensated by combining it with a concave lens. A short-sighted person therefore wears concave spectacles.

Fig. 69.

In old people, and sometimes in young ones, the ciliary muscle may not be strong enough to accommodate the lens for forming images of near objects on the retina. Eyes which suffer from this defect are *long-sighted* or *hypermetropic*. Since the lens is not convex enough it may be assisted by the use of another convex lens. Long-sighted people therefore wear convex glasses.

There is another and probably commoner disease of the eye known as *Astigmatism*. This is due to a want of symmetry in the eye.

In Fig. 70 are shown some astigmatic lenses. It will be seen that a vertical section of such a lens is like that of an ordinary lens, while a horizontal section would be like that of a plane sheet of glass. The eye is never so asymmetric as this. The horizontal and vertical sections would only show a slight difference in curvature, but it is clear that an astigmatic lens would not focus horizontal and vertical lines at the same distance. In Fig. 71 an attempt has been made to show the kind of image which would be given by an astigmatic lens when the object is a piece of gauze. The horizontal lines in the image are all nearer to the lens than the vertical lines.

When an eye suffers from this defect it is readily remedied by the use of an astigmatic lens. If the faces of the lenses of Fig. 70 had equal curvature their combination would give a plane sheet of glass. It is thus possible to find an astigmatic lens of the right degree of astigmatism and to set it at such an angle in front of an astigmatic eye that the eye sees normally.

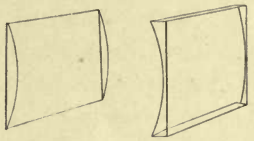


Fig. 70.

§ 64. *Judging Distances. Binocular Vision.*—We have seen that, in order to focus correctly an object at any given distance, a certain strain must be put on the ciliary muscle in order that the lens may have the proper focal length.

The extent of this strain gives us some notion of the distance of

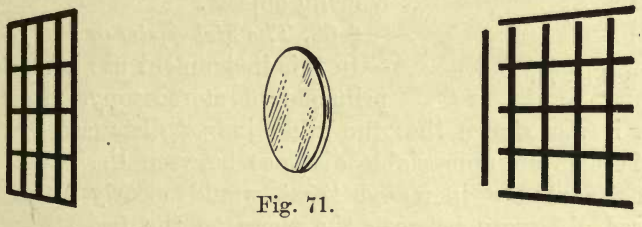


Fig. 71.

the object, but only a rough notion. Any one who has tried the experiment of shutting or bandaging one eye, and then trying to reach out to an object hanging in the air, knows how poor a guide to distance is the strain of the ciliary muscle.

If, however, both eyes are used it is a comparatively easy matter to judge the distance of an object. This is because the two eyes do not have quite the same view of an object. Suppose that R and L (Fig. 72) are two eyes looking at an object $C_1$. The view which each eye gets is indicated by dotted lines. The right eye sees more of the right-hand side of the object, and the left eye more of the left-hand side. Now if the object is removed to $C_2$ the difference between the two views is not so marked. The difference between the views seen by the eyes gives an appearance of solidity to the mental effect of binocular (or two-eyed) vision ; but it does more than this, because the

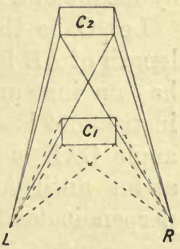



Fig. 72.

notion of solidity becomes less marked as the distance of the object increases, and this flattening of the view gives an idea of distance.

In the case of a row of objects at various distances, such as are shown in section by A, B, C, D, E, in Fig. 73, the distances are judged not only by the stereoscopic flattening of the objects taken singly, but also by the fact that the right eye sees the objects in the order ABCDE from left to right, while the left eye sees them in the reverse order, EDCBA from left to right. Stereoscopic effect depends, therefore, partly on the appearance of an object itself and partly on its apparent relationship to neighbouring objects.

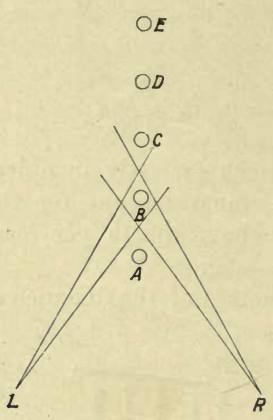


Fig. 73.

§ 65. *The Mekometer or Range-Finder.* —In this instrument use is made of the principle of stereoscopy. In the last section it was shown that the mind judges distances by the fact that there is an appreciable distance between the eyes. If this distance could be increased there would clearly be a greatly increased difference between the views on the two retinæ. This is the essential principle of the British Army Range-Finder. Here two men are used as the two eyes and the distance between them is fixed by means of a string of fixed length.*

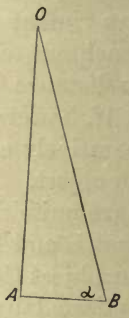


Fig. 74.

Let O be the object and A and B the men. The length of AB is the length of the string. Each man has an instrument at his end of the string. A's instrument is so arranged that he can tell when the angle OAB is a right angle. Both A and B move about until A's instrument indicates that AB is perpendicular to AO. B's instrument indicates the angle OBA ($= \alpha$) and since $\frac{AO}{AB} = \tan \alpha$ $\therefore$ AO $=$ AB $\tan \alpha$. On the scale of B's instrument are marked the values of AB $\tan \alpha$, and hence AO can be read off directly.

* The theoretical principle of the new Marindin instrument is the same, though the practical details are different.

§ 66. *The Compound Microscope.*—In the ordinary compound microscope two convex lenses are employed. The small lens of short focal length near the object is called the *objective.* The larger lens at the other end of the instrument is the *eye-piece.*

When an object (O, Fig. 75) is placed at a distance from the objective greater than its focal distance a real inverted image $I_1$ is formed inside the microscope (*see* § 56). The eye-piece is placed at such a distance from this image ($I_1$) that a virtual enlarged image ($I_2$) is formed.

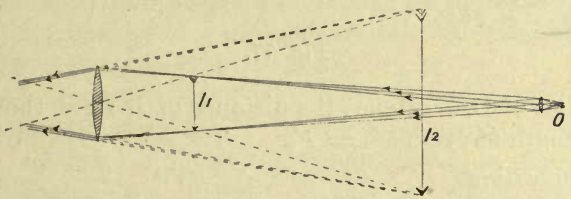



Fig. 75.

§ 67. *The Astronomical Telescope.*—In the astronomical telescope the same arrangement of lenses is employed. The sizes and focal lengths of the lenses are, however, different. The object to be viewed is extremely distant, and the image ($I_1$, Fig. 76) formed by the objective will therefore be at the principal focus of the objective.

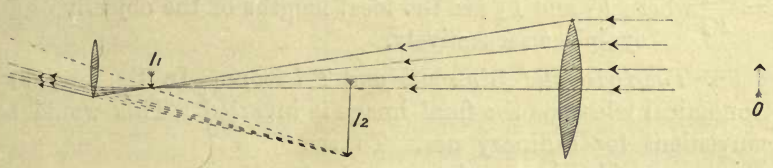



Fig. 76.

This image is viewed by the eye-piece which forms a magnified virtual image $I_2$. This final image ($I_2$) is seen to be upside down as regards the original object O.

The magnification

$$= \frac{\text{size of } I_2}{\text{size of O}} = \frac{\text{size of } I_2}{\text{size of } I_1} \times \frac{\text{size of } I_1}{\text{size of O}}.$$

From § 52 we know that

$$\frac{\text{size of } I_2}{\text{size of } I_1} = \frac{\text{distance of } I_2 \text{ from eye-piece}}{\text{distance of } I_1 \text{ from eye-piece}}$$

and

$$\frac{\text{size of } I_1}{\text{size of O}} = \frac{\text{distance of } I_1 \text{ from objective}}{\text{distance of O from objective}}.$$

When the telescope is focussed for a normal eye there is no strain on the ciliary muscle (§ 62), and consequently the eye-lens is fairly flat and the distance of $I_1$ is very great, and may be taken as

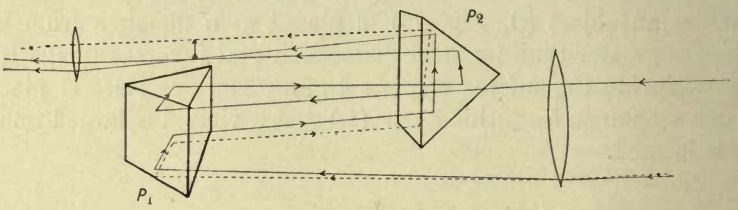



Fig. 77.

equal to that of O. Hence the distance of $I_1$ from the eye-piece = focal length of eye-piece = $F_2$.

$\therefore$ magnification

$$= \frac{\text{distance of } I_2 \text{ from eye-piece}}{\text{distance of } I_1 \text{ from eye-piece}} \times \frac{\text{distance of } I_1 \text{ from objective}}{\text{distance of O}}$$

$$= \frac{\text{distance of } I_2 \text{ from eye-piece}}{\text{distance of O from objective}} \times \frac{\text{distance of } I_1 \text{ from objective}}{\text{distance of } I_1 \text{ from eye-piece}}$$

$$= \frac{\text{distance of O}}{\text{distance of O}} \times \frac{F_1}{F_2}$$

$= \frac{F_1}{F_2}$ where $F_1$ and $F_2$ are the focal lengths of the objective and eye-piece respectively.

§ 68. *The Prismatic Binocular and Telescope.*—In the ordinary astronomical telescope the final image is inverted. This would be inconvenient for ordinary use. To see the stage upside down when using a binocular in a theatre would, for instance, be uncanny. The prismatic telescope gets over this difficulty; it is shown in section in Fig. 77. The rays after refraction at the objective are reflected by total reflection (§ 40) inside a right-angled prism $P_1$. Another view of this prism is shown in Fig. 78; it will be seen that the angle of incidence on the reflecting face is 45°, and that if total reflection is to take place this angle (45°) must be greater than the critical angle, *i.e.*, *sin* 45° $> \mu$. Since *sin* 45° = 0·7071, the refractive index from air

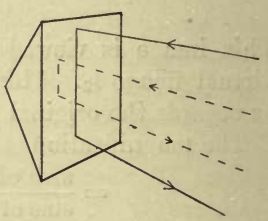


Fig. 78.

to glass must be $> \frac{1}{\cdot 7071}$, *i.e.*, $> 1{\cdot}41$, so that no very special kind of glass is required.

In Fig. 78 two rays are traced through the prism. (One is drawn in a broken line for clearness.) It will be noticed that the rays are reversed in position by reflection. The prism $P_1$ in the telescope will therefore turn the image round from right to left. The second prism $P_2$ acts in a similar manner, but, being set at right angles to $P_1$, its effect is to turn the image upside down.

The combined effect of the two prisms is, therefore, completely to invert the image; as the image is naturally an inverted one, this re-inversion makes it finally erect.

If a telescope were made with the same lenses, but without the reflecting prisms, it would be nearly three times as long as the prismatic telescope. Since the magnification is great when an objective of long focal distance is used it will be seen that the system of prisms enables high magnification to be attained in a short instrument.

*Examples.*

1. Describe the construction and use of the magic lantern. Give a careful drawing of a pencil of rays passing from the slide to the screen. (Army 01.)

2. Compare the optical properties of the eye with those of a photographic camera.

A man cannot see any object distinctly which is less than 4 ft. from him; what spectacles does he require to enable him to read a paper 1 ft. from his eyes? (Camb. M.B. 04.)

3. What are the functions, in the eye, of (1) the cornea, (2) the lens? For what kind of defect in the eye is a cylindrical lens used? (Camb. M.B. 05.)

4. Explain by means of a diagram the action of spectacles in enabling a short-sighted person to see distant objects. (Camb. Loc. Sen. 05.)

5. Explain how a person who is short-sighted is enabled by using a lens to see things at a distance. Of what sort and of what focal length should the lens be if the person can only see things clearly up to 2 ft. away? (Lond. Inter. Sci. 04.)

6 What do you understand by astigmatism as applied to the eye? What kind of lens should be used to remedy the defect of an eye that is both short-sighted and astigmatic? (Camb. M.B. 05.)

7. Give a brief general account of the eye as an optical instrument. Suppose a short-sighted eye can see an object clearly only when it is placed at a distance not exceeding 8 ins., what kind of lens should be used, and of what power, in order that if placed close to the eye it would enable objects that are 48 ins. away to be clearly seen? (Lond. Inter. Sci. 01.)

8. Explain how two suitable convex lenses may be combined to form a

microscope, and draw a diagram illustrating the mode of formation of the image seen by the eye.

Explain why, when a slide is moved in any direction across the stage of a microscope, the image moves in the opposite direction. (Camb. M.B. 08.)

9. The object glass and the eye-piece of a compound microscope are each 1 in. in focal length, and the distance between them is 9 ins. If the image is formed at the distance of distinct vision, 10 ins., where must the object be placed? (Oxf. Loc. Sen. 05.)

10. Find by a geometrical construction, or otherwise, how the image given by a convex lens of an object on its axis changes in size and position as the object is moved up to the lens from a distance.

A simple astronomical telescope has an object glass of 10 ins. focal length, and an eye-piece of 1 in. focal length, and is in adjustment for a distant. object. Draw as accurately as you can the path of some *one* ray from the object to the eye of the observer, and explain your construction. (Oxf. Prelim. and C. (3) 06.)

11. The lenses of a telescope have focal lengths of 1 ft. and 1 in. respectively. The telescope is directed towards a spire 1 mile away, and the distance between the lenses is adjusted until the image of the spire is distinct Give a diagram showing the passage through the telescope to the eye of the rays from the point of the spire.

Why would the rays from a still more distant spire give a distinct image without further adjustment of the telescope? (Matric. 08.)

12. How would you arrange two convex lenses of 15 cms. focal length so as to give an erect image equal in size to the object? (Camb. Loc. Jun.)

13. Describe a microscope. What is the essential difference between a microscope and a telescope? (Camb. Loc. Jun. 07.)

14. Describe a simple telescope formed of two lenses. What advantage is gained by having a large object glass? (Camb. Loc. Sen. 07.)

15. On what do the following properties in a simple astronomical telescope depend?

1. Magnifying power.
2. Field of view.
3. Brightness of image. (Army 00.)

16. Explain the terms *total reflexion* and *critical angle.* How would you exhibit total reflexion from the surface of (1) water, (2) a glass prism? What is the refractive index of a substance for which the critical angle with respect to air is 45°? (Matric. 08.)

17. Explain what is meant by total internal reflection. A glass prism (index of refraction 1·5) has one angle 90°, the other two equal to 45°; if the prism is placed between the eye and a distant object, describe in a general way the changes in the apparent position of the object as the prism is turned round its axis. (Army 04.)

18. Describe an experiment to illustrate the phenomenon of total reflexion.

Light is incident normally on one of the faces containing the right angle of a right-angled isosceles prism. What is the least refractive index the material of the prism can have in order that the light may be totally reflected from the hypotenuse of the prism? (Camb. Loc. Sen. 07.)

## CHAPTER V

### SPECTROSCOPY.—COLOUR

§ 69. *The Spectroscope.*—In § 44 it was shown that the amount of deviation produced in a ray of light refracted through a prism depends on the angle of incidence of the ray. It is true that when the prism is so placed that the deviation is a minimum there is very little difference produced by a slight change in the angle of incidence.

If we calculate the deviation corresponding to angles of incidence in the neighbourhood of 49° for a prism whose angle is 60° and whose refractive index is 1·5, we obtain the following results :—

| | | | | | |
|---|---|---|---|---|---|
| $i$ . . . | 48° 30′ | 48° 40′ | 48° 50′ | 49° 0′ | 49° 10′ |
| D . . . | 37° 11′ | 37° 11′ | 37° 12′ | 37° 12¼′ | 37° 12½′ |

It is clear that if we wish to make accurate observations of the deviations we must ensure that all the rays incident on the prism will have the same angle of incidence. In other words, all the rays incident on the prism must be parallel.

In the spectroscope this is done by means of the *collimator* C (Fig. 79A). A horizontal section is shown in Fig. 79B. Rays from the lamp L enter the collimator by a slit S, which is shown also in Fig. 80, and thence pass to a lens at the other end of the collimator. This lens is convex, and is placed at such a distance from the slit that the image of the slit is at a very great distance, and the rays emerging from the lens are therefore parallel. The rays then fall on the prism P (Figs. 79A and 79B), and are thereby bent so as to fall on to the objective of an astronomical telescope T (*see* § 67). Since all the rays entering the prism are parallel, they will all be bent to the same extent, and will therefore emerge parallel. The image formed by the objective of the telescope will therefore be at its principal focus.

The telescope is mounted on an axis, so that by pushing it with the hand it can be moved in a circle, round the prism. On the central upright of the instrument is a scale graduated in degrees and half degrees. A pointer and vernier (*see* § 3, "Introductory") are

attached to the telescope so that its angular position can be read to the nearest minute of arc. Besides this, arrangements are also

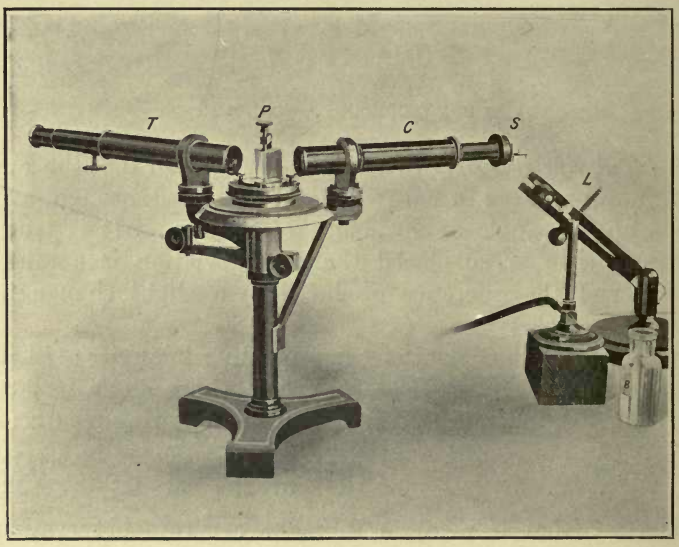



Fig. 79A.

usually made for clamping the telescope and central upright together by means of a screw, so that the telescope can no longer

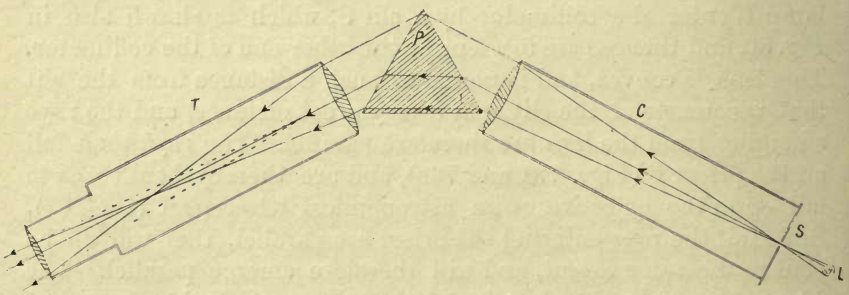



Fig. 79B.

be moved by pushing with the hand. A second screw is so arranged that the telescope can be slowly moved over a small arc when it is clamped, and so accurately adjusted to any desired position.

§ 70. *To Set up and Adjust the Spectroscope.*—When the collimator is properly arranged the light emerges from it as a parallel beam. When this is so the distance between the slit and lens must be equal to the focal length of the lens. It is not, however, practical to measure the focal length of the lens and then to set the slit accurately at the required distance; recourse is therefore had to an indirect method.

In the telescope is a fine wire set at about the position where the image of the slit is formed by the objective.

Fig. 80.

The eye-piece end of the telescope is shown in section in Fig. 81, where this wire is indicated by W. It will be seen that the eye-piece can be pulled in and out of the tube so that its distance from W is altered, and that the instrument is so made that the eye-piece and W can be pulled in and out of the tube simultaneously. The first adjustment is provided so that W can be placed at such a distance from the lens that its image is at a comfortable distance from the observer's eye. When this has been done the prism is removed from the table and the telescope is turned round so that a clear view of some distant object is obtained. By sliding the eye-piece and wire in and out a position can be found where the image of the distant object coincides with the wire. When this is so the wire and the image of the distant object will not appear to shift relatively to one another when the observer's eye is moved from side to side (*see* § 19).

Fig. 81.

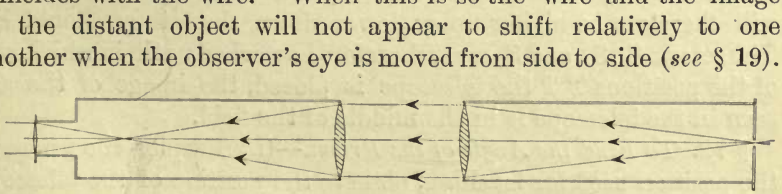


Fig. 82.

We now know that the telescope is so adjusted that light from a distant object, *i.e.*, a parallel beam, is brought to a focus at the wire.

Turn the spectroscope so that a bright light falls on the slit, and then turn the telescope so that light from the collimator may fall on it as in Fig. 82.

If the image of the slit formed by the collimator lens is at a very great distance, then we know that the telescope objective will bring the light from it to a focus at the wire. But if the image

in the telescope is not on the wire, then the light from the collimator is not coming from a very distant image, and the distance between the slit and the collimator lens must be altered. The slit is therefore worked in and out until its image in the telescope coincides with the wire.

We have now arranged the collimator so that it gives a parallel beam of light, and the telescope so that this beam can be readily viewed. The next operation is :—

§ 71. *To Set the Prism Upright on its Table.*—Looked down on from above, the prism table has the appearance of Fig. 83. Three screws $S_1$, $S_2$, $S_3$ are provided for levelling the table; there is also a clamp for holding the prism in its place. Set the prism in the position shown, with its angle towards the collimator C. Some of the light from the collimator falling on the prism faces will be refracted into the prism, but some will be reflected. If the telescope is placed in either of the positions shown (T T) this reflected light will enter it, and an image of the slit will be visible to the observer. If, however, the prism faces are not upright, these reflected beams will not only be bent to one side, but will be bent upwards or downwards by reflection. The images seen will therefore be higher up or lower down than they should be. The screws $S_1$, $S_2$, and $S_3$ must therefore be worked until, in whichever of the positions T T the telescope is placed, the image of the slit seen in the telescope is in the middle of the field.

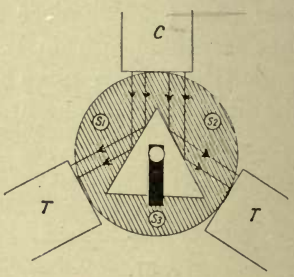


Fig. 83.

§ 72. *To Find the Angle of the Prism.*—(i.) Keeping the prism in the position used for levelling, observe the position of the telescope when the image of the slit is exactly on the cross-wire. There are two positions of the telescope in which this will be the case (T T, Fig. 83). The difference between the two readings will give the angle between the reflected rays. Since the incident rays are parallel, this angle will be double the angle of the prism (§ 18).

*Example.*

| | |
|---|---|
| 1st reading . . . | 265° 41′ |
| 2nd reading . . . | 145° 38′ |
| Difference . . . | 120° 3′ |

Therefore angle of prism = 60° 1′ 30″.

(ii.) Clamp the telescope firmly at right angles to the collimator. Turn the prism table until the image of the slit seen by reflection from one face (AB, Fig. 84) of the prism lies exactly on the cross-wire. Read the position of the pointer and vernier attached to the prism table (not the one on the telescope). Turn the prism table until the image reflected from the face AC is on the cross-wire. Again read the position of the pointer on the prism table.

The angle through which the prism is turned will be the angle DAC (Fig. 84), *i.e.*, $180° - \angle BAC$. Hence $\angle BAC$ = angle of prism = 180° — difference between readings.

*Example.*

| | |
|---|---|
| 1st reading . . . | 210° 16′ |
| 2nd reading . . . | 90° 18′ |
| Difference . . | 119° 58′ |

$\therefore$ angle of prism = 60° 2′.

§ 73. *Observations on Light Deviated by Refraction through the Prism.*—Arrange the prism and telescope in the position shown in Fig. 79. On observing the image of the slit in the telescope a broad band of colour will be seen. Further examination will show that one end of this band is red, the other blue, and that between these lie other colours gradually blending one into the other. It will also be found that the red is least deviated, the blue most. This band of colour is called the *Spectrum*. There are two explanations of this appearance which suggest themselves: either that the prism has in some way made the colours, or that the original beam of light contained all these colours and that the prism has separated them. The second explanation is the more probable, and is the one most usually accepted.

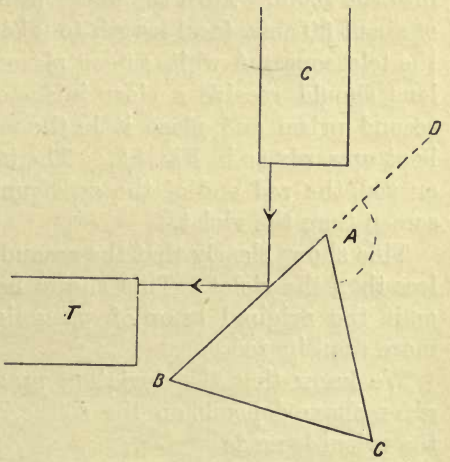



Fig. 84.

§ 74. *Light is Composite.*—If we accept this suggestion, that a beam of white light is in reality a mixture of all colours and that the prism is able to separate these, several experiments suggest themselves to test our theory.

Take a piece of ruby glass such as photographers use for their dark-room lanterns and place this in front of the slit of the spectroscope. Most of the spectrum will disappear, only a small dark red portion remaining (*see* Fig. 89). Place the ruby glass between the collimator and prism or between the prism and telescope. In all cases the effect is the same. The ruby glass removes all kinds of light from a white beam except the red rays, and is equally effective before and after the prism has split the beam.

Various other experiments of the same kind might be suggested; all would lead to the same result. The next question is, How does the prism split up a beam of light? The following experiment shows this.

§ 75. *Different Refrangibility of Rays of Different Colours.*—Take the spectroscope into a dark room and place a lamp in front of the slit. The lamp must be so screened that very little light comes out into the room. Turn the telescope aside and fix up a convex lens of about 30 cms. focal length to take the place of the objective of the telescope. A white screen placed at the principal focus of this lens should receive a clear image of the spectrum on it. Take a second prism and place it in the path of the beam so that this is bent upwards as in Fig. 85. The image on the screen will now be curved, the red end of the spectrum being decidedly lower on the screen than the violet.

This shows clearly that the second prism has bent the red rays less than the violet. This shows how the first prism was able to split the original beam of white light by bending its violet rays more than its red ones.

We know that the deviation produced in a ray by a prism of a given shape depends on the refractive index of the glass of which the prism is made.

It is therefore obvious that if we wish to use the spectroscope to measure the refractive index of the glass of a prism we must so arrange matters that light of some particular colour only is used.

§ 76. *The Sodium Flame.—Monochromatic Light.*—Put the spectroscope together again and set the telescope in the position of Fig. 79. Instead of an ordinary lamp in front of the slit use a Bunsen burner in which a piece of pumice soaked in a sodium

carbonate solution is placed. The flame of the Bunsen is in this way coloured a somewhat ghastly yellow. When the image in the telescope is looked at it will be found to be a sharp yellow image of the slit. It is clear that this " sodium flame " gives only light of one colour. By using a piece of ruby glass in § 74 we obtained red light only; but this did not give a sharp red image of the slit, since it was a mixture of various kinds of red light. The yellow light from

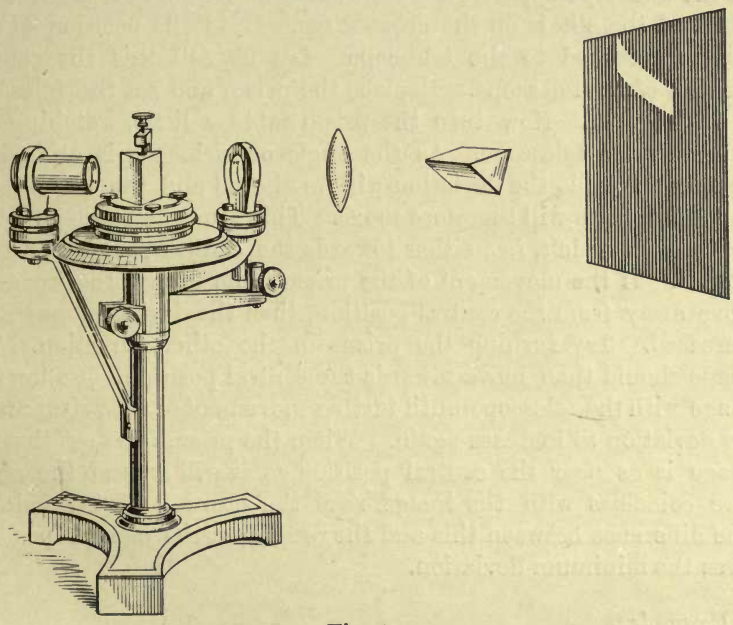


Fig. 85.

a sodium flame is not a mixture of various kinds of yellow light (or at any rate of many kinds), but is practically one pure yellow.

On the basis of the wave theory, the difference between the colours of various kinds of light is caused by differences in the number of vibrations taking place in the wave per second. The deepest visible red rays make about $4 \times 10^{14}$ vibrations per second, while those in the end of the visible violet make about $8 \times 10^{14}$ vibrations.*

The mixture of red rays obtained by means of ruby glass would

* It will be noticed that the eye is sensitive to about an octave of light vibrations, while the ear is sensitive to about eight or nine octaves of sound vibrations.

have vibration numbers varying between $4{\cdot}16 \times 10^{14}$ and $4{\cdot}64 \times 10^{14}$ per second, while the mixture given by a sodium flame would lie between $5{\cdot}096 \times 10^{14}$ and $5{\cdot}101 \times 10^{14}$ vibrations per second. For practical purposes, therefore, a sodium flame may be considered to give light of one kind only. A sodium flame is *monochromatic.*

§ 77. *To Find the Least Deviation of a Ray of Sodium Light Producible by a Given Prism.*—Remove the prism from its table and set the telescope opposite the collimator as in Fig. 82, so that the image of the slit is on the cross-wires. Read the position of the pointer attached to the telescope. Let us call this the central position of the telescope. Replace the prism and set the telescope as in Fig. 79. Now turn the prism table a little, watching the image in the telescope. As the angle of incidence is altered by turning the table the deviation will be altered and the image seen in the telescope will therefore move. This movement will be either to the right or left, *i.e.*, either towards the central position or away from it. If the movement of the prism table causes the image to move away from the central position, then the deviation has been increased. Try turning the prism in the other direction. The image should then move towards the central position. Follow the image with the telescope until further movement of the prism causes the deviation to increase again. When the prism is so set that the image is as near the central position as it will go and the cross-wire coincides with the image, read the position of the pointer. The difference between this and the reading at the central position gives the minimum deviation.

*Example.*

| | |
|---|---|
| Reading at central position . . . | 200° 30′ |
| Reading in position of minimum deviation | 258° 52′ |
| Deviation . . . . . | 58° 22′ |

To obtain a correct result the prism and telescope should be turned so that an observation is made on the other side of the central position. The position on the telescope will now indicate about 142° when the prism is in the position of least deviation. This will give a second determination of the deviation, which should agree with the first.

§ 78. *To Calculate the Refractive Index of the Prism for Sodium Light.*—In § 43 it was shown that the deviation is least when the

ray passes symmetrically through the prism, and from this was deduced the formula

$$\mu = \frac{\sin\frac{D + A}{2}}{\sin\frac{A}{2}}.$$

We can use the values obtained above to calculate $\mu$ for sodium light for the glass of which the prism is made.

$$\mu = \frac{\sin\frac{58^\circ\ 22' + 60^\circ\ 1'\ 30''}{2}}{\sin\frac{60^\circ\ 1'\ 30''}{2}}$$

$$= \frac{\sin 59^\circ\ 11'\ 45''}{\sin 30^\circ\ 0'\ 45''} = \frac{\cdot 8589}{\cdot 5002}$$

$$= 1\cdot 7172.$$

§ 79. *Flame Spectra.*—When other sodium salts are used in place of sodium carbonate, the same yellow image is always observed in the telescope; the only noticeable difference being that as some sodium salts are more volatile than others, the image is brighter in some cases than in others. We may therefore conclude that this kind of yellow light is characteristic of sodium. With a good spectroscope it is readily seen that the sodium spectrum consists of two yellow lines very close together, so close in fact that for many cases they may be considered as one single line.

If a potassium salt is placed in the flame the spectrum will be seen to contain a red line, and if the flame used is hot enough there will also be a violet line. Other salts give other kinds of spectra and these are so far characteristic, that we may make the following generalizations:—

(i.) The spectra of no two metals contain the same lines.

(ii.) By examining the spectrum given by a salt it is possible to say what metal or metals are present.

In Fig. 86 a map of the flame spectra is given for the commonest elements.

§ 80. *Gas Spectra.*—When an electric discharge from an induction coil (§ 111, E.C.), is passed through a tube containing a rarefied gas, the tube glows with a flickering light. The colour of the light is characteristic of the gas in the tube. Nitrogen gives a rose-red light, carbon monoxide a pale blue, and hydrogen a bright red. A

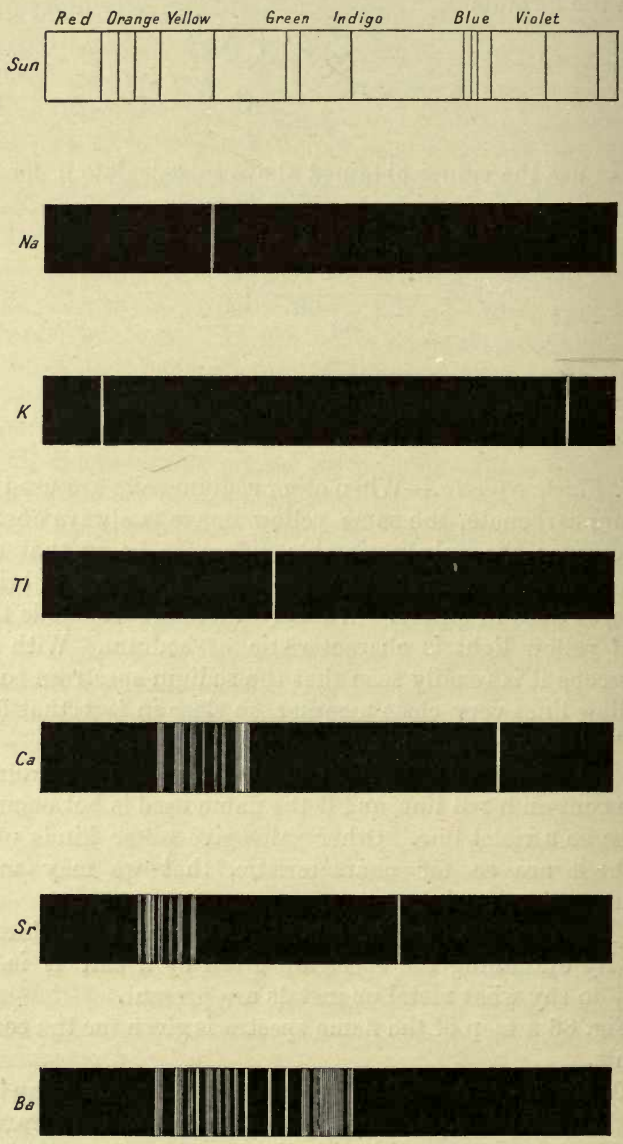

Red
Orange
Yellow
Green
Indigo
Blue
Violet
Sun
Na
K
Tl
Ca
Sr
Ba

Fig. 86.

good form of tube for observing the spectra of these discharges is shown in Fig. 87; when the narrow central part of the tube is placed in front of the slit of the spectroscope it is found that each gas has a characteristic spectrum. These spectra are usually more complicated than the flame spectra of § 79; hydrogen and helium give only four or five bright lines each, but nitrogen gives a spectrum containing an enormous number of lines, all of them bright and many of them crowded together (*see* Fig. 88). Carbon monoxide gives a very lovely arrangement of bands which have a fluted appearance.

Fig. 87.

§ 81. *Absorption Spectra.*—In § 74 an experiment was described in which a piece of ruby glass cut off a large portion of the spectrum leaving only a portion of the red. We may describe this residual red as the absorption spectrum of ruby glass. If a piece of blue glass is used the absorption spectrum is found to contain blue, green, and some yellow light, the whole of the red* and orange and part of the yellow being cut off by the glass. In this way we can prove that every coloured transparent object has its own absorption spectrum. These spectra are quite unlike the flame spectra described in the last paragraph, since they contain broad bands of colour with diffuse edges and not clear-cut lines. If the light reflected from coloured objects, such as coloured paper or cloth, is examined, it is found that here again we have absorption spectra.

Take a piece of blue paper and examine it by the light obtained from a lamp surrounded by ruby glass; the paper will appear almost black. Since the paper absorbs red, orange, and yellow light it can only reflect blue and green light. The ruby glass cuts off blue, green, and yellow light, and so leaves no light of any kind which the blue paper can reflect. No light is reflected from the paper, and it consequently appears black.

We can thus see that the colour of ordinary objects depends on their absorption spectrum and on the kind of light supplied to them. This explains the well-known changes of colour of objects apparently caused by changes in the nature of the light used. Many blues

* Ordinary cobalt glass transmits a small band in the deep red.

appear green by gaslight; the lobelia flower is blue by daylight and mauve by lamplight.

§ 82. *Pigment Colours.*—No ordinary dyed object or painted object has an absorption spectrum consisting of one colour only. Ordinary blue pigment, such as prussian blue, reflects both blue and green light. Now gaslight contains less blue light in proportion than does daylight. If the spectra of daylight and gaslight are looked at side by side, and if the gas flame is made so bright that the yellow parts of the spectra appear equally bright, the blue and violet of the gas spectrum will appear very dim beside the blue and violet of the sun spectrum. Consequently blue paint will appear green by gaslight, since the gas lamp supplies very little blue

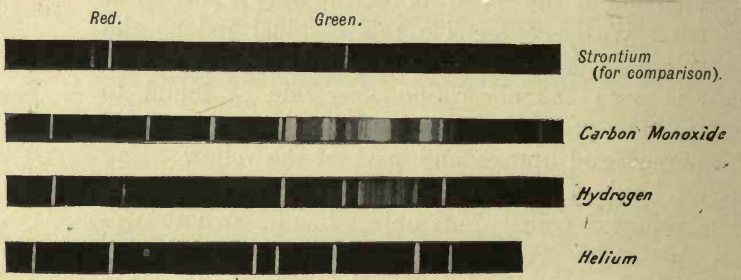



Fig. 88.

light to be reflected. A blue pigment, which reflects very little green, such as cobalt, simply appears as a rather darker blue by gaslight.

We may regard the green light reflected by prussian blue as an impurity, and may therefore say that cobalt is a purer blue than prussian blue. Similar remarks apply to other pigments. Lemon yellow is a very pure colour, and reflects very little green. Gamboge reflects both yellow and green.

When gamboge and prussian blue are mixed the one pigment absorbs all the blue light, the other all the yellow, and the only light which both can reflect is green. Consequently a painter says that when "blue and yellow are mixed we get green."

If, however, we mix cobalt and lemon yellow, neither reflects much green light, and each absorbs the light reflected by the other. Hence their mixture does not give a proper green, but an obvious mixture of blue and yellow.

The same sort of analysis of the impurities in other pigments will enable us to see what will be the result of mixing them. But

in all cases we find that the effect of mixing is not really to mix the colours, but to show up impurities.

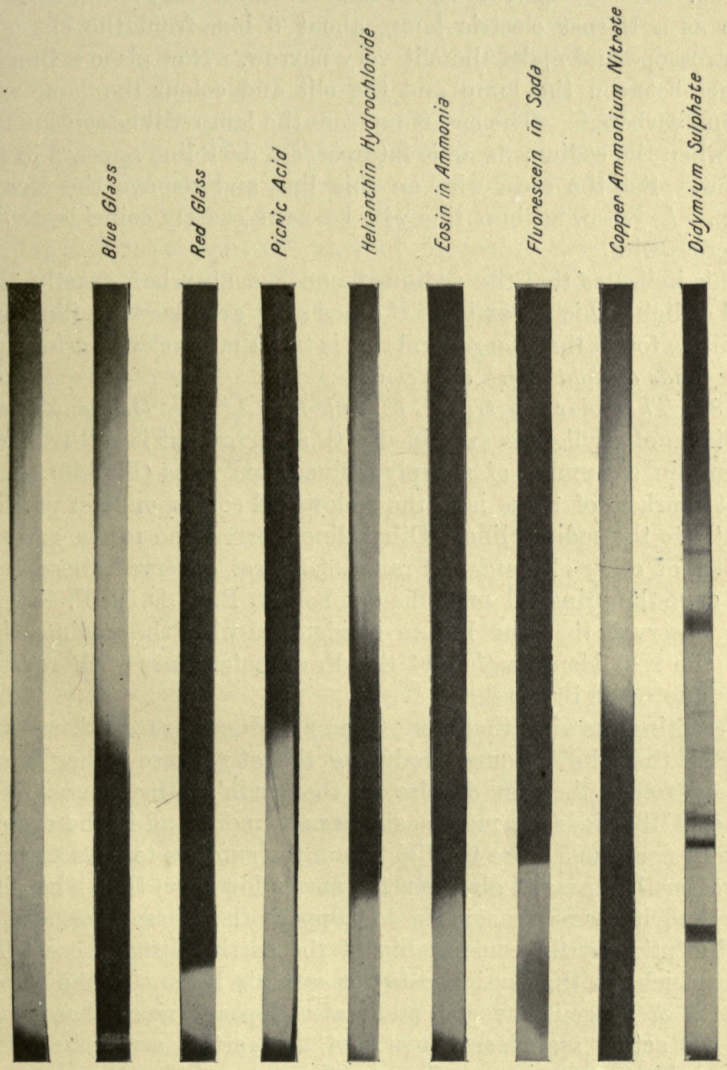



Fig. 89.

The absorption spectra of pigments are small bands in the spectrum, and consequently by mixing pigments corresponding to neighbouring portions of the spectrum, we obtain a mixture of

which the absorption spectrum is represented by the overlapping portions of the original spectra.

§ 83. *Reversal of the Spectrum.*—Place a very bright lamp, such as a Nernst electric lamp, about 6 ins. from the slit of a spectroscope and make the slit very narrow. Now place a Bunsen burner between the lamp and the slit and colour the flame with sodium carbonate. The spectrum from the lamp will be continuous, but when the sodium flame is interposed a dark line appears in the yellow. Set the cross-wire on this line and remove the Nernst lamp. A yellow sodium line will be seen exactly coincident with the cross-wire.

This indicates that the sodium flame has absorbed exactly that kind of light which it emits. If other salts are placed in the flame it will be found that the general rule is true that *flames absorb exactly those kinds of light which they emit.*

§ 84. *The Solar Spectrum. Fraunhofer's Lines. Helium.*—If the spectrum of daylight is examined with a narrow slit it will be found to contain a number of sharply defined dark lines (Fig. 86). The most marked of these is in the yellow and corresponds in position exactly to the sodium line. Other lines correspond to the reversal spectra of other elements. Fraunhofer first observed these lines and gave the principal ones the symbols A, B, C, D, E, F, G. He also observed that the D line is identical with the sodium line. But the real identification of the Fraunhofer lines with reversal lines was made by Kirchoff.

Accepting the view that these lines are due to reversal, we may suppose that the vapours producing the effects are either immediately round the sun or else in the earth's atmosphere. Now some of the lines are more easily seen at morn and in the evening than at noon, and since the light from the sun has to pass through a greater thickness of air when the sun is low down than when it is overhead it seems reasonable to suppose that these lines are due to absorption by the constituents of the earth's atmosphere. The lines which are thus assigned to our own air belong to the known spectra of oxygen, nitrogen, and water vapour; since these gases are the actual main constituents of the earth's atmosphere it is reasonable to suppose that they are the cause of these lines.

Other lines, however, do not correspond to gases known to exist in our atmosphere, nor are they affected by the position of the sun in the sky. These lines must therefore be caused by absorption in

the sun's chromosphere. By examining them astronomers have been able to find what elements are present round the sun and thus to make an analysis of the chromosphere.

In 1891 Sir Norman Lockyer announced that, after examining the Fraunhofer lines carefully, he found that some of them could not be assigned to any element known to us, and must be due to an unknown substance which he called Helium. Three years later Lord Rayleigh and Sir William Ramsay found that on heating a mineral called Cleveite, a gas was evolved which gave the spectrum which Lockyer had assigned to helium (Fig. 88). We may thus say that Sir Norman was able by means of the spectroscope to discover a new element ninety-two millions of miles away.

§ 85. *Application of Doppler's Principle to Moving Stars.*—In § 76 it was pointed out that the colour of a ray of light depends on the number of vibrations per second. We have now learnt that colour and position in the spectrum are the same, and may therefore measure the vibration number of a wave by its position in the spectrum. If a star is moving rapidly towards the earth the light waves sent out from it in any instant will be crowded forward on those sent out in the previous instant, and the vibration number of the waves received by us will be increased. Similarly the light waves from a retreating star will have a diminished frequency (compare § 32, s.). As a result of such motion the positions of the lines in the spectra of certain heavenly bodies are slightly shifted in the spectrum, and from the amount of shift it is possible to calculate the rate of motion causing the shift.

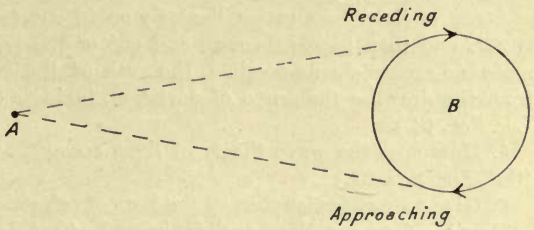


Fig. 90.

The principle may also be applied to rotating bodies (such as B, Fig. 90) since one edge is approaching towards A and the other receding from A. In this way is has been possible to confirm previous estimates as to the rate of rotation of the sun.

*Examples.*

1. You are given a prism, a screen with a fine slit in it, and three converging lenses; two of the lenses are of focal lengths 15 cms., the other is of focal length 5 cms. Draw a diagram (to scale) to show how you would arrange these to form a spectroscope. (Camb. Loc. Jun. 07.)

2. Show that if a plane mirror be turned through an angle $\theta$ about an axis in its own plane, a ray reflected in a plane perpendicular to the axis of rotation will be turned through an angle $2\theta$.

Give a careful sketch of the arrangement of a lamp, slit, lens, and scale, by means of which the image of the slit formed by the lens and reflected by a plane mirror may be thrown on to the scale. Trace in your sketch the course of a pencil of the rays. (Lond. Inter. Sci. 01.)

3. State the laws of reflexion, and explain how they are applied in the optical determination of the angle of a prism. (Camb. Loc. Sen. 06.)

4. What is meant by the statement that the critical angle for glass is 41° 48′?

A ray of light falls perpendicularly on to one face of a prism whose angle is 30°, and then passes through and out of the prism. Trace in a drawing, made as accurately as possible, the path of the emergent ray, and estimate from the drawing the angle of deviation, taking $\mu$ (air into glass) $= \frac{3}{2}$. (Oxf. Loc. Sen. 04.)

5. How does the wave theory of light account for (*a*) colour, (*b*) reflexion? (Oxf. Loc. Sen. 04.)

6. Give a short explanation of the laws of refraction on the wave theory.

How is colour explained on this theory? (Oxf. Loc. Sen. 07.)

7. Prove that a ray of light undergoes no deviation by passage through a transparent plate, and state, with reasons, whether the same is true for a prism.

Explain an arrangement by which you could show that the deviation of a given coloured light by a given prism cannot be less than a certain amount. (Oxf. Prelim. and C. (3) 07.)

8. Explain how you would adjust a spectrometer and use it to measure the angle of minimum deviation for light passing through a prism. (Camb. Spec. 07.)

9. Describe fully how you would determine the index of refraction of the glass of a prism, given the formula

$$\mu = \frac{\sin \frac{1}{2}(\mathrm{D} + i)}{\sin \frac{1}{2} i}.$$

In an experiment $i = 60°$, $\mathrm{D} = 39° 46'$; calculate the value of $\mu$. (Army 05.)

10. Describe a method of determining the index of refraction of a liquid by the aid of a hollow glass prism. Is it necessary to take account of the refractive index of the glass? (Camb. Loc. Sen. 07.)

11. Light from a slit is allowed to fall on a prism. State and explain what may be observed when the slit is illuminated with (*a*) sodium flame, (*b*) white light. (Camb. Loc. Jun. 06.)

12. A spectroscope is in adjustment, with the prism in the position of minimum deviation for sodium light. Draw the paths of rays of blue and red

light through the collimator, the prism, and the telescope. (Oxf. Prelim. and C. (3) 05.)

13. Explain carefully how the deviation of a parallel beam of light by a glass prism varies with the angle of incidence of the beam on the first surface of the prism. How does it depend for a particular angle of incidence on the angle of the prism, on the refractive index of the glass and on the colour of the light? (Lond. Inter. Sci. 06.)

14. What experimental arrangements would you make in order to obtain "a pure spectrum"? How far is the expression "a pure spectrum" justified? (Lond. Inter. Sci. 05.)

15. What is meant by (1) a continuous spectrum, (2) an absorption spectrum? How would you test whether red light coming from two different sources was of the same colour? (Camb. M.B. 04.)

16. A snow-covered landscape when viewed through a piece of red glass appears red. How does the glass produce such a result? How would you verify your explanation by experiment?

Red light is found to penetrate a fog more effectively than light of any other colour. This being so, would the penetrating power of an arc lamp be increased by passing its rays through red glass? (Matric. 04.)

17. To what are the colours of objects as ordinarily seen due—for instance, red glass, blood, red cloth, red copper? (Lond. Inter. Sci. 04.)

18. How would you arrange experiments to show, with an adjusted spectroscope,

(*a*) an emission spectrum,

(*b*) the absorption spectrum of red ink?

Why must the slit of a spectroscope be narrow? (Oxf. Prelim and C. (3) 03.)

19. Give a special explanation of Fraunhofer's lines in the solar spectrum; and describe an experiment to verify the explanation. (Oxf. Loc. Sen. 05.)

20. In what ways can we by means of a spectroscope determine some of the chemical elements in the luminous stars? Explain carefully the principle on which this method of analysis is based. (Army 00.)

## CHAPTER VI

### DISJECTA MEMBRA

§ 86. *Correction of Lenses for Colour. Achromatic Lenses.*—We know that the focal length of a lens depends on the refractive index of the glass of which it is made. The refractive index, however, depends on the kind of light, and therefore the focal length of a simple lens is different for different kinds of light. If the real image of a distant white object formed by a common convex lens is looked at it will appear to have coloured edges. Since red light is less bent than violet the red image will clearly be further off than the violet image, and therefore the red image will be larger than the violet. The coloured edge should therefore have the red outside and the various other colours of the spectrum in order inside.

It will be easier to see how this colour effect is got rid of in good lenses if we consider first how a prism can be made to deviate a ray of light without producing a spectrum.

If we take a prism of crown glass having an angle of 60°, on which a parallel beam of white light falls with an angle of incidence of 49°, the blue rays near the blue hydrogen line will be deviated through an angle of 38° 51′, yellow rays near the sodium line through an angle of 37° 56′, and red rays near the red potassium through an angle of 37° 28′. There will therefore be an angle of 38° 51′ − 37° 28′ = 1° 23′ between the potassium and the hydrogen lines. If this beam is now sent through a prism of flint glass having an angle of 18° set as in Fig. 91, these three rays will be deviated in the opposite directions through angles of 18° 23′, 17° 26′, and 17° 0′ respectively. The total deviations of the three lines will therefore be 20° 28′, 20° 30′, and 20° 28′. They emerge, in fact, practically parallel.

If we compare the refractive indices of crown and flint glass for the three rays considered above we can see the explanation of these effects.

| | Crown. | Flint. | Ratio. |
|---|---|---|---|
| $\mu$ for red potassium line . . | 1·5033 | 1·7218 | 1·145 |
| $\mu$ for yellow sodium line . . | 1·5086 | 1·7371 | 1·152 |
| $\mu$ for blue hydrogen line . . | 1·5190 | 1·7721 | 1·167 |

In the third column of the table are given the values of the ratios of $\mu$ for the two kinds of glass. We see that the change in the refractive index brought about by a change in the method of manufacture of the glass is not the same at all parts of the spectrum. It is therefore possible by choosing two prisms of different kinds of glass and of different angles to deviate a beam of light with very little separation into colours.

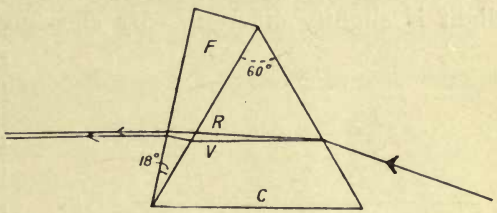



Fig. 91.

It is also possible so to combine two prisms that the beam is separated into colours with very little deviation. If we combine a 60° crown glass prism with a flint glass prism whose angle is 44° set so as to reverse the deviation produced by the crown glass prism, we find the following values for the deviations :—

| | *Deviation by 1st prism. | Deviation by 2nd prism. | Total deviation. |
|---|---|---|---|
| Hydrogen blue . . | 38° 48′ | 40° 50′ | 2° 2′ |
| Sodium yellow . . | 38° 11′ | 38° 53′ | 42′ |
| Potassium red . . | 37° 41′ | 37° 59′ | 18′ |

When a lens is regarded as a collection of prism sections as in § 45, it is easily seen how a lens composed of a convex portion of one kind of glass combined with a concave part of another kind of glass could give an achromatic compound lens.

* For a ray incident at 45° on the first prism.

We might combine a crown glass convex lens, having focal lengths for the hydrogen, sodium, and potassium lines — 19·27, — 19·66, and — 19·87 cms., with a flint glass concave lens for which the corresponding focal lengths are + 41·44, + 43·40, and + 44·33. By the formula of § 58 we could then calculate the combined focal lengths.

$$-\frac{1}{19{\cdot}27}+\frac{1}{41{\cdot}44}=-\frac{1}{36{\cdot}01}\;;\quad -\frac{1}{19{\cdot}66}+\frac{1}{43{\cdot}40}=-\frac{1}{35{\cdot}95}\;;$$

$$\text{and}\quad -\frac{1}{19{\cdot}87}+\frac{1}{44{\cdot}33}=-\frac{1}{36{\cdot}01}.$$

The focal lengths for red and blue are equal, but that for yellow light is slightly different. By choosing more suitable kinds of glass better compensation could have been obtained; it is, however, improbable that any pair of glasses has yet been found to give a perfect equalisation of focal length throughout the spectrum.

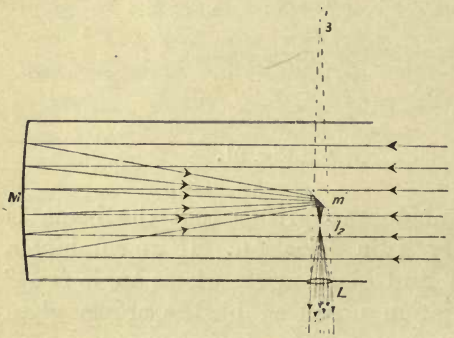



Fig. 92.

§ 87. *Newton's Reflecting Telescope.* — In Newton's time it was not known that by combining different kinds of glass a lens could be made which would give an image free from colour. Newton therefore invented a telescope in which only one small lens was employed. This instrument is shown diagrammatically in Fig. 92. M is a concave mirror of long focal length; this forms an image of a distant object. Since the laws of reflection are the same for all kinds of light, this image is free from colour. Before the light reaches this image it falls on a plane mirror *m* set at 45° to the axis of the telescope, and is so reflected as to give an image $I_2$. This image is viewed by means of a small convex lens L placed so close to $I_2$ as to give a virtual magnified image $I_3$.

§ 88. *Herschel's Telescope.*—Herschel by tilting the concave mirror of Newton's telescope was able to dispense with the small mirror. His telescope is shown diagrammatically in Fig. 93.

§ 89. *Galileo's Telescope.*—In § 45 it was shown that the effect

of a concave lens is to make the beam passing through it more divergent. If the beam before refraction is convergent, then the lens may make it either less convergent, parallel, or divergent. The lens formula

$$\frac{1}{V} = \frac{U + F}{UF}$$

indicates the conditions under which each of these three effects will result. If the original beam is convergent it is going *to* an image, and therefore $U$ is $-^{ve}$. $F$ for a concave lens is $+^{ve}$. Hence,

$V$ is $-^{ve}$ if $U + F$ is $+^{ve}$, *i.e.*, $F$ is numerically greater than $U$.
$V$ is $\infty$ if $U + F$ is zero, *i.e.*, $F$ is numerically equal to $U$.
$V$ is $+^{ve}$ if $U + F$ is $-^{ve}$, *i.e.*, $F$ is numerically less than $U$.

In other words the refracted beam will diverge from a virtual image on the $+^{ve}$ side if the light before refraction is converging to an image at a greater distance from the lens than its principal focus.

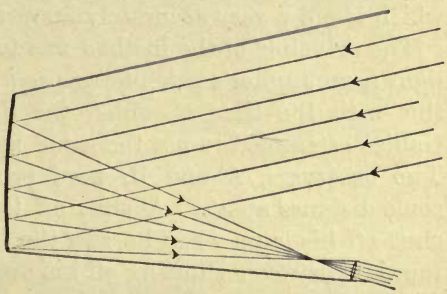


Fig. 93.

This principle is made use of in Galileo's telescope (Fig. 94). A large convex objective forms an inverted image; before the light reaches this image it falls on a small concave lens so placed that a virtual image is formed on the same side of the lens as the original object, *i.e.*, on the opposite side to the first image.

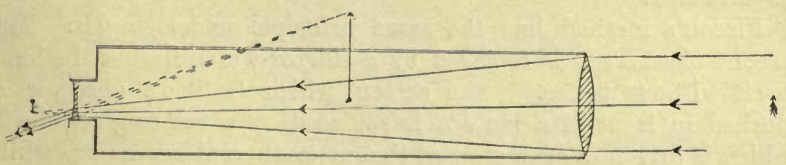


Fig. 94.

Since the last image is on the opposite side of the eye-piece to the first image it will be the opposite way up to the first

image (§ 57), that is, the same way up as the original object. This fact makes Galileo's telescope far more convenient than the astronomical telescope for ordinary use.

The magnification produced by this telescope may be shown to be equal to the ratio of the focal lengths of the objective and eye-piece just as in the case of the astronomical telescope (§ 67). Since the length of the instrument is the numerical difference between these focal lengths, Galileo's telescope is shorter for the same magnification than the astronomical telescope, the length of which equals the sum of the focal lengths of the lenses.

§ 90. *The Velocity of Light in Air. Fizeau's Method.*—In § 2 it was shown that the velocity of light can be estimated by means of astronomical observations. While this method has the advantage that the times to be measured are long and can be accurately observed, it has the disadvantage that the distance over which the passage of the beam is observed is the diameter of the earth's orbit, which is not a very accurately known quantity.

The principle of the method was to observe the time at which the light from Jupiter's satellite was *seen* to be cut off and to compare this with the time at which the light was *known* to be cut off. Galileo attempted to use the same method on the earth's surface. Two observers, A and B, were provided with lamps and placed some distance apart. A shut off the light from his lamp and B shut off his lamp when he saw the eclipse of A's lamp. A noted the time between shutting off his own lamp and seeing the light of B's lamp disappear. Knowing as we do that light will travel thousands of miles in a fraction of a second, we can easily see that the time taken in putting dark slides in front of the lanterns would be so immense compared with the time which Galileo wanted to observe that no satisfactory result could be hoped for from the experiments.

Fizeau's method has the same principle underlying it. The second observer B is replaced by a mirror which flashes the light back without pause. The screen which Galileo's observer A pushed in front of a lantern is replaced by a rotating cogwheel, which is driven at such a speed that a tooth replaces a gap while the light passes from A to B and back.

The apparatus used is shown in Fig. 95. The part A consists of a bright source of light S. Near this is a convex lens arranged to form a real image of S. Before the light reaches this image it falls

on a sheet of glass placed at 45° to the beam, part of which is therefore reflected through a right angle towards B. The image of S will therefore be in front of this sheet of glass. A cogged wheel W is so placed that this image either falls between two of its teeth or on a tooth. If the image is between two teeth the light will travel on till it reaches the lens $L_1$ where it is rendered parallel. This parallel beam then travels to the distant part of the apparatus B, which consists of a convex lens $L_2$ and a concave mirror. Parallel light falling on $L_2$ is brought to a focus on the surface of the mirror and reflected back to $L_2$, which renders it parallel again. After passing $L_1$ the beam converges to the wheel and then diverges until it reaches the sheet of glass. Here some of it is reflected back to S, but some will pass through to the observer's eye at E.

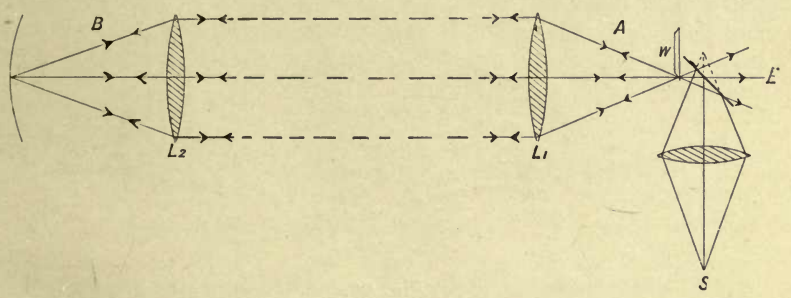



Fig. 95.

Now suppose that the cogged wheel is rotated. For a short time there is a gap and light can pass from A to B and be reflected back. If by the time the light begins to arrive at W the next gap has not got into position, the beginning part of the return flash will be cut off, and the observer will only see that part of the flash which gets back after the gap has begun to let light through. If the wheel is rotated somewhat faster a gap will have got into position in time to let through the returning beam which passed through the previous gap on its outward journey. By rotating the wheel faster still, a flash going out by gap No. 1 will not get back in time for gap No. 2, but will be stopped by the succeeding tooth. No light will therefore reach the observer. If the speed at which the wheel rotates is known, then the time taken for a gap to replace a tooth or a gap is known. But this is the time taken for the light to go from A to B and back.

Fizeau's wheel had 720 teeth; the distance from A to B was 8,633 metres, and it was found in one experiment that least light was visible at E when the wheel made 12·6 revolutions per second.

Now at this speed a tooth replaces at gap in $\frac{1}{2}\frac{1}{12{\cdot}6}\frac{1}{720}$ secs.

Light therefore goes $2 \times 8{,}633$ metres in $\frac{1}{2}\frac{1}{12{\cdot}6}\frac{1}{720}$ secs., or

$2 \times 8{\cdot}633 \times 2 \times 12{\cdot}6 \times 720 = 313{,}000$ kilometres per second.

More accurate experiments made by Cornu with an improved apparatus gave an average value of 300,000 kilometres per second.

§ 91. *Foucault's Method.*—In § 16 it was shown that a beam of light reflected from a mirror is deflected when the mirror is rotated, and that the angle through which the reflected beam is turned is

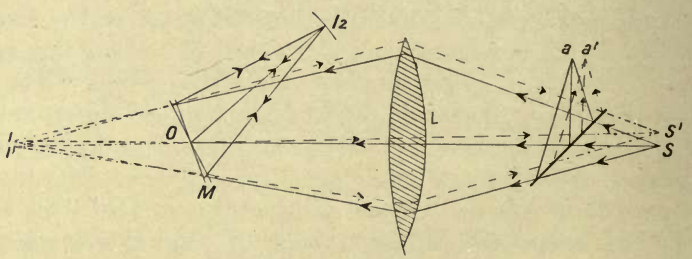



Fig. 96.

twice the angle through which the mirror is turned. If, therefore, we can measure the angle through which a beam of light is deflected by turning a mirror we can readily calculate the angle through which the mirror is turned. Suppose that the mirror is mounted on an axle so that it can be rapidly rotated at a known speed, then we can calculate the time taken for the mirror to turn through the observed angle. It is this method of estimating very short intervals of time which is used in Foucault's method for finding the velocity of light.

Fig. 96 shows the apparatus used. Light from a brightly illuminated slit S passes through a plane sheet of glass and falls on the lens L. An image of S is therefore formed at I. Before the light reaches I it is reflected at a plane mirror M which can be rotated, about an axis through O. The reflected beam converges to $I_2$ on the surface of a concave mirror of which the centre of curvature is at O. The beam is thus reflected back, and when M is stationary

will go to S. Some of the returning beam is, however, reflected by the glass sheet to a point $a$, where the observer's eye may be placed.

Now suppose that the mirror is rotated at such a speed that while the beam passes from M to $I_2$ and back the reflecting surface changes from the position shown by one of the lines in Fig. 96 to that shown by the other. The reflected beam will no longer go to S, but to S′, and hence the light reflected by the glass sheet will go to $a'$ instead of $a$.

The observer can measure the distance between $a$ and $a'$ accurately. This will clearly be the same as that between S and S′. This was observed to be 0·7 mm.

Suppose that while the light goes from O to $I_2$ and back the mirror M turns through an angle $\theta$. Then the beam will be turned through an angle $2\theta$ and will appear to come from a point I′ near I such that

$$\frac{II'}{IO} \text{ radians} = 2\theta.$$

But we know from similar triangles (see page 118) that

$$\frac{II'}{IL} = \frac{SS'}{SL},$$

since the rays IS and I′S′ pass through the centre of the lens.

In Foucault's experiments IL = 20 metres and SL = 2·1 metres, and therefore

$$II' = \frac{0{\cdot}7 \times 20}{2{\cdot}1} = 7 \text{ mm.}$$

$$\therefore 2\theta = \frac{7}{IO} \text{ radians} = \frac{7}{2\pi \times IO} \text{ of a complete revolution,}$$

$$\text{or } \theta = \frac{7}{4\pi . IO} \text{ of a revolution.}$$

If the mirror made $n$ revolutions per second it would take $\frac{7}{4\pi n . IO}$ secs. to turn through $\theta$. But in this time the light goes from O to $I_2$ and back, a distance of IO × 2 metres.

In one second, therefore, light travels $\frac{2 \times IO \times 4\pi n \times IO}{7}$ metres. From the values actually observed by Foucault the velocity of light in air appears to be 298,000 kilometres per second.

§ 92. *Velocity of Light in other Media.*—In § 34 it was shown that the wave theory of light indicates that the velocity of light in any medium B should be given by

$$\text{Velocity in B} = \frac{\text{velocity in A}}{\text{refractive index from A to B}}.$$

Now it is comparatively easy to fill the space $I_2O$ in Foucault's apparatus with a liquid or solid of known refractive index, and so to test this formula. Foucault himself was content to show that the velocity is less in water than in air, but Michelson made actual measurements of the velocity not only in water, but in carbon bisulphide. In the case of water it was found that

$$\frac{\text{Velocity in air}}{\text{Velocity in water}} = 1{\cdot}330, \text{ whereas } \mu = 1{\cdot}333.$$

The slight difference may be ascribed to experimental errors.

§ 93. *Colour Perception and Colour Mixing. Maxwell's Colour Box.*—In § 82 it was pointed out that the result of mixing pigments is not the same as that of mixing colours. Clerk Maxwell devised an arrangement of prisms by which the prismatic colours could be mixed and viewed by the eye. The following table shows some of his results compared with the results of mixing pigments :—

| | Mixed Colours. | Mixed Pigments. |
|---|---|---|
| Red . . . } Blue . . . } | Purple | Purple |
| Red . . . } Green . . } | Yellow | Brown |
| Blue . . . } Yellow . . } | Pink | Green |

Now these results confirm a theory which had been put forward by Young, who suggested that the eye really only sees three colours—probably red, green, and blue. It may be that some of the nerves of the retina are only affected by red, others only by blue, and others only by green; or it may be that each nerve is capable of receiving three kinds of sensation and only three, but this is not probable.

When some other kind of light falls on the eye it affects two sets of nerves. For instance, yellow light affects both the red-sensitive

and the green-sensitive nerves; if these are equally excited the eye judges the yellow to be fairly pure. If the red-sensitive nerves are more highly excited than the green-sensitive, then we call the resulting sensation orange-yellow or orange.

We may represent the theory diagrammatically by a triangle (Fig. 97) at the corners of which are the three primary sensations. The mixed sensations lie along the sides and inside the triangle. Thus, purple is a mixture of the red and blue sensations and therefore lies on the side joining the red and blue corners. Yellow lies on the side joining red and green. Pink lies somewhere on a line joining blue and yellow, and therefore excites all three sensations in various degrees.

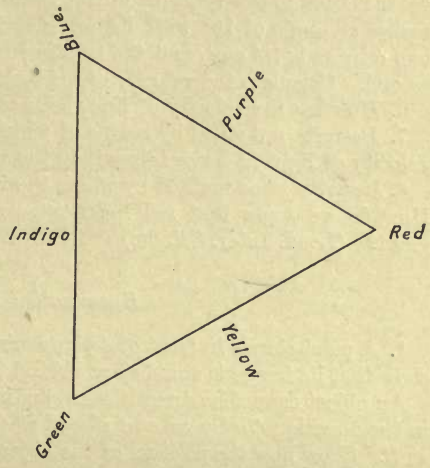



Fig. 97.

§ 94. *Colour Blindness.*—This view is confirmed by a study of certain defective eyes. There are people in whose eyes one of the colour sensations is lacking or very feeble. The sensation most usually wanting in such cases is red. When this is the case only the blue and green sensations remain and all colours will appear as varieties of blue, indigo, and green. Purple appears blue, yellow appears green, and pink is a shade of indigo. Very often the sense of red is not lacking, but only weaker than in other folk. This has the effect of shifting all the colours in our triangle over towards the line joining blue and green. It is probable that no two people have the three sets of nerves equally sensitive, and this would account for the preference which some people show for colours which others consider hideous.

*Examples.*

1. Give a drawing showing how a ray of white light is dispersed by a prism. The image of a very small source of white light is thrown on to a screen by a convex lens. If the screen is brought nearer the lens, the spot of light is

enlarged to a round patch, and its edge is coloured. How do you explain this? (Lond. Inter. Sci. 02.)

2. Show that it is possible to make an achromatic prism or lens with two kinds of glass. (Lond. Inter. Sci. 03.)

3. How may (*a*) two convex lenses, (*b*) a convex and a concave lens be combined to form a telescope? Will the image viewed be erect or inverted? (Camb. Spec. 05.)

4. Give a diagram showing how an image is formed by a concave lens.

A concave and a convex lens each 6 ins. focal length are placed 18 ins. apart. An object is placed 9 ins. from the convex lens on the side furthest from the concave lens, and is looked at through both lenses. What is the position of the image that is seen? (Army 02.)

5. A beam of light is reflected by a rotating mirror on to a fixed mirror which sends it back to the rotating mirror from which it is again reflected and then makes an angle of 18° with its original direction. The distance between the two mirrors is $10^6$ cms. and the rotating mirror is making 375 revolutions per second. Calculate the velocity of the light. (Lond. Inter. Sci. 06.)

6. How has the velocity of light been ascertained? (Oxf. Loc. Sen. 04.)

7. Describe a method by means of which it has been shown directly that the velocity of light in water is less than the velocity in air. (Camb. Spec. 07.)

8. Describe some method by which the velocity of light has been determined. How do we know that red light travels more quickly than blue light inside glass? (Lond. Inter. Sci. 06.)

*Examination Paper I.*

1. A point stands in front of a plane mirror; assuming the laws of reflexion, show that its image is equidistant behind the mirror.

An object cannot be directly seen, but its image in a plane mirror $AB$ is seen in the direction $CD$ by an eye placed at $C$ and in the direction $EF$ by an eye at $E$. Show how the position of the object may be determined. (Oxf. Prelim. and C. (3) 04.)

2. Why does water appear to be less deep than it really is? What will be the apparent depth of a pond 4 ft. deep when looked at vertically ($\mu$ for water $= \frac{4}{3}$)? (Camb. Loc. Jun. 05.)

3. Explain the theory of some method of comparing the illuminating powers of two sources of light. What difficulty does the experiment present? (Camb. Loc. Sen.)

4. Define the "principal focus" and the "optical centre" of a lens. Show that the latter is a fixed point for a given lens.

Explain clearly the advantage of a parabolic mirror over a spherical one, when it is required to project a parallel beam of light. (Oxf. Loc. Sen. 05.)

5. Describe clearly how you would determine the focal length of a convex lens.

At what distance from a convex lens of focal length 8 ins. would you place an object in order to produce a real image of this object magnified 3 times (linear)? (Army 03.)

6. Obtain a formula connecting the distances of an object and its image

from a convex spherical reflecting surface, and calculate the ratio of the size of the image to that of the object. (Camb. Loc. Sen. 06.)

7. How would you carry out an experiment to show that the illumination of a surface, by a small source of light, varies inversely as the square of its distance from that source? Describe some form of simple photometer. (Lond. Inter. Sci. 01.)

8. To what is the colour of bodies, as ordinarily seen, to be attributed? (Lond. Inter. Sci. 05.)

9. What is meant by the position of minimum deviation for a prism? Why is this position used in determining the index of refraction of the material of a prism? (Camb. Loc. Sen. 07.)

*Examination Paper II.*

1. What is the difference between a real and a virtual image of an object formed by a spherical mirror? Illustrate by diagrams.

A pin 3 cms. long is placed with its middle point at a distance of 1·5 metres from a concave spherical mirror whose radius is 50 cms.; find the position and size of the image formed. (Oxf. Loc. Jun. 08.)

2. What are the laws of the refraction of light? Show how to find the apparent position of an object when seen through a thick plate of glass. (Camb. M.B. 05.)

3. A reflecting galvanometer is fitted with a small spherical mirror of 1 yd. radius. Show how you would arrange the lamp and scale, which are to be at the same distance from the mirror, to obtain a well-defined spot of light on the scale. Calculate the angle of deflection of the galvanometer when the spot of light moves along 4 ins. from the centre of the scale. (Army 05.)

4. Explain what is meant by total internal reflection and give some illustrations of it. (Army 00.)

5. A jar is placed 3 ft. in front of a lens of focal length 18 ins. Where, and how, must a flower be placed behind the lens in order to appear to stand naturally in the jar to an eye suitably placed? Give a diagram, showing the position of the observer. (Army 04.)

6. How would you show on general grounds that the intensity of illumination at a point depends on the square of its distance from the source of light?

An electric glow lamp when placed 3 yds. from a Joly photometric screen gives equal intensity of illumination with a Carcel lamp (9·5 British standard candles) placed 6 ft. 3½ ins. from the screen. Calculate the candle-power of the lamp. (Army 05.)

7. Distinguish between dispersion and deviation.

How is it possible to get the one without the other? (Camb. Spec. 05.)

8. Give a short account of the nature of the spectra observed when the slit of a spectroscope is illuminated by (i.) sunlight, (ii.) light from a luminous gas flame, (iii.) light from a Bunsen flame in which a salt of sodium is held, (iv.) light, from a luminous flame, which has passed through a coloured solution, *e.g.*, potassium permanganate. (Camb. Spec. 07.)

9. Describe and explain an optical method of measuring the magnitude of the angle between the faces of a prism. (Oxf. Prelim. and C. (3) 04.)

*Examination Paper III.*

1. Light coming in a fixed direction is reflected from a mirror. Show that when the mirror is turned through an angle $\delta$ the reflected ray is twisted through the angle $2\delta$. How is this principle applied in the sextant to measure the angle between distant objects? (Army 04.)

2. Explain how to find the refractive index of a liquid for a given monochromatic light by the method of total internal reflexion. (Camb. Loc. Sen. 06.)

3. A ray of homogeneous light is incident at an angle of 40° on one face of a prism. Find the direction of the ray in the prism. [The index of refraction of glass is 1·5.] (Camb. Loc. Jun.)

4. What is meant by the focal length of (*a*) a mirror, (*b*) a lens?

Determine graphically the position of the image formed of an object placed 30 cms. from a reflecting surface of which the radius of curvature is 40 cms. (Camb. Loc. Jun. 05.)

5. Show how the focal length of a convex lens depends on the curvatures of its surfaces. (Lond. Inter. Sci. 05.)

6. Explain Foucault's method of determining the velocity of light by means of a rotating mirror. (Oxf. Prelim. and C. (3) 06.)

7. Show why a convex lens gives rise to colour effects.

The image of a small bright flame is formed by a convex lens on a screen; explain with the aid of diagrams how the edge of the image is coloured according as the screen is (*a*) rather too near to the lens or (*b*) rather too far away. (Oxf. Loc. Sen. 07.)

8. Give a brief description of a spectroscope, and describe and account for the different spectra seen when the slit is illuminated by (*a*) sodium light, (*b*) an ordinary luminous gas flame, (*c*) sunlight. (Oxf. Prelim. and C. (3) 07.)

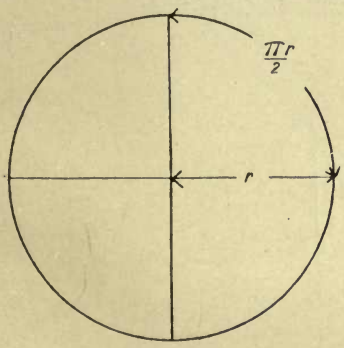
9. Describe a method for measuring the refractive index of a solid. (Oxf. Loc. Sen. 04.)

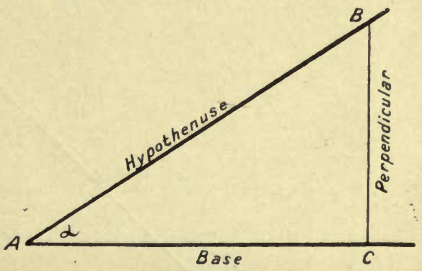
10. Describe the eye as an optical instrument. What kind of lens would you use to remedy the defect of a long-sighted eye? (Camb. M.B. 03.)

# APPENDIX II

## NOTES ON TRIGONOMETRY

§ 1.—On page 7 (Vol. I.) it was pointed out that angles may be measured as fractions of a right angle. Now a right angle cuts off $\frac{1}{4}$ of the circumference of a circle, and therefore the arc subtended by a right angle is $\frac{1}{4}2\pi r = \frac{\pi}{2}$ of the radius. An angle of 1 degree will therefore subtend an arc $\frac{\pi r}{180}$. Such a method of measuring angles involves the use of a circle, and a method could clearly be devised in which use was made of straight lines only; this method is outlined




below, and deals with what are generally known as trigonometrical ratios. Suppose $\alpha$ is the angle whose trigonometrical ratios must be obtained. From any point in one of the bounding lines drop a perpendicular on the other bounding line. In this way the triangle ABC is obtained, having a right angle at C. The ratios of the sides of this triangle are independent of its size (Euc. VI. 2), and are in fact properties of the angles BAC and ABC.

If the sides AB, BC, CA, are called the hypothenuse, perpendicular, and base respectively, then the ratio,

$$\frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{CB}}{\text{AC}} \text{ is called the tangent of A} = \mathbf{\tan \alpha}$$

$$\frac{\text{Perpendicular}}{\text{Hypothenuse}} = \frac{\text{CB}}{\text{AB}} \quad \text{,,} \quad \text{sine} \quad \text{,,} = \mathbf{\sin \alpha}$$

$$\frac{\text{Base}}{\text{Hypothenuse}} = \frac{\text{AC}}{\text{AB}} \quad \text{,,} \quad \text{cosine} \quad \text{,,} = \mathbf{\cos \alpha}$$

$$\frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{AC}}{\text{CB}} \quad \text{,,} \quad \text{cotangent} \quad \text{,,} = \mathbf{\cot \alpha}$$

§ 2.—

$$\frac{\sin \alpha}{\cos \alpha} = \frac{CB}{AB} \div \frac{AC}{AB} = \frac{CB}{AC} = \tan \alpha$$

$$\frac{\cos \alpha}{\sin \alpha} = \frac{AC}{AB} \div \frac{CB}{AB} = \frac{AC}{CB} = \cot \alpha$$

§ 3.—In the triangle ABC the angles BAC and ABC are together equal to a right angle, or $\angle ABC = 90° - \alpha$.

It is therefore clear that

$$\sin (90° - \alpha) = \frac{AC}{AB} = \cos \alpha \qquad \cos (90° - \alpha) = \frac{BC}{AB} = \sin \alpha$$

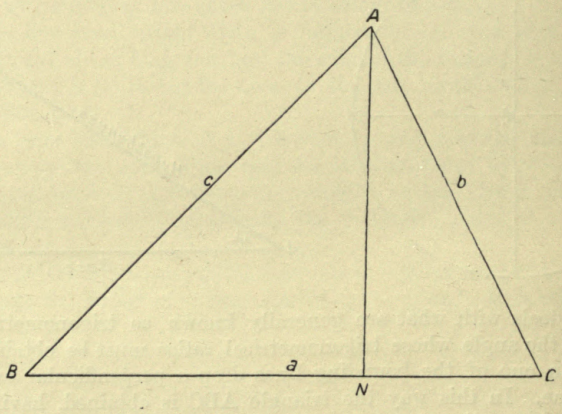
$$\tan (90° - \alpha) = \frac{AC}{CB} = \cot \alpha \qquad \cot (90° - \alpha) = \frac{CB}{AC} = \tan \alpha.$$

§ 4.—Since the angle at C is a right angle,

$$AC^2 + CB^2 = AB^2.$$

Hence

$$\left(\frac{AC}{AB}\right)^2 + \left(\frac{CB}{AB}\right)^2 = 1, \text{ or } \mathbf{\cos^2 \alpha + \sin^2 \alpha = 1}.$$




§ 5.—Consider the △ ABC in which none of the angles are right angles. Let the sides BC, CA, AB, be equal to $a$, $b$, $c$, respectively.

From A drop a perpendicular AN or BC. Then

$$\frac{\sin B}{\sin C} = \frac{AN}{c} \div \frac{AN}{b} = \frac{b}{c} \text{ or } \frac{\sin B}{b} = \frac{\sin C}{c}.$$

By dropping a perpendicular from B on AC it may be similarly shown that $\frac{\sin C}{c} = \frac{\sin A}{a}$, and hence

$$\mathbf{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}.$$

§ 6.—(i.) To find the value of $sin\ (\alpha + \beta)$.

Let $\angle CAB = \alpha$ and $\angle DAB = \beta$. Through B draw CBD perpendicular to AB; and from C draw CE at right angles to AD. Then $sin\ (\alpha + \beta) = EC \div AC$.

Now the area of a $\triangle = \frac{1}{2}$ base $\times$ altitude.

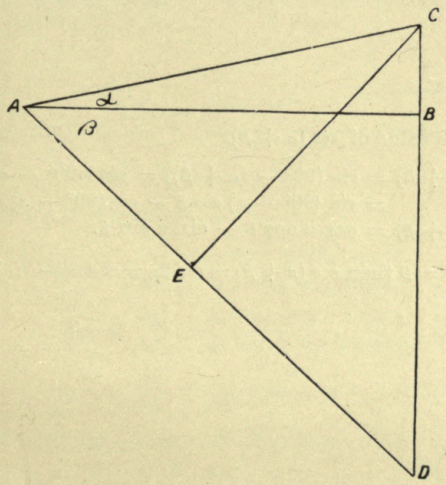
$$\therefore \text{Area of } ACD = \tfrac{1}{2}\, EC \,.\, AD = \tfrac{1}{2}\, AB \,.\, CD$$

$$\therefore EC = \frac{AB \,.\, CD}{AD} = \frac{AB \,.\, (CB + BD)}{AD} = \frac{CB \,.\, AB}{AD} + \frac{AB \,.\, BD}{AD}$$

$$\therefore sin\ (\alpha + \beta) = \frac{EC}{AC} = \frac{CB \,.\, AB}{AC \,.\, AD} + \frac{AB \,.\, BD}{AC \,.\, AD}$$

$$\mathbf{sin\ (\alpha + \beta) = sin\ \alpha\ cos\ \beta + cos\ \alpha\ sin\ \beta.}$$

Corollary.—If $\alpha = \beta$ then $sin\ (\alpha + \beta) = \mathbf{sin\ 2\alpha = 2\ sin\ \alpha\ cos\ \alpha.}$




(ii.) To find the value of $cos\ (\alpha - \beta)$.

$$cos\ (\alpha - \beta) = sin\ (90° - (\alpha - \beta)) = sin\ ((90° - \alpha) + \beta)$$
$$= sin\ (90° - \alpha)\ cos\ \beta + cos\ (90° - \alpha)\ sin\ \beta.$$

$$\therefore \mathbf{cos\ (\alpha - \beta) = cos\ \alpha\ cos\ \beta + sin\ \alpha\ sin\ \beta.}$$

(iii.) To find the value of $sin\ (\alpha - \beta)$.

Let angle $CAD = \alpha$ and $\angle CAB = \beta$. (Fig. on page 170.)

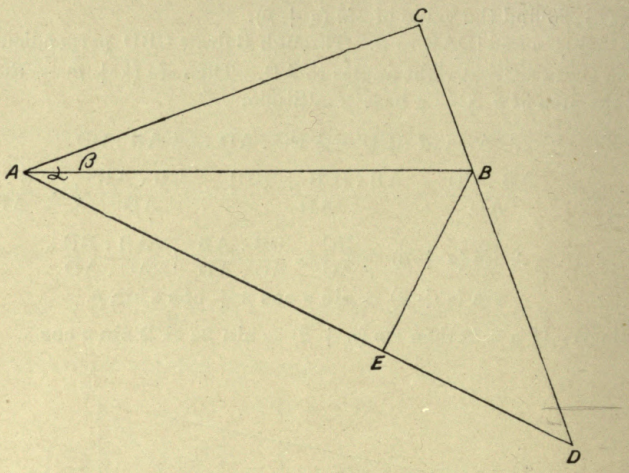
Draw CBD perpendicular to AC, and from B drop a perpendicular BE on AD.

Area of $\triangle ABD = \frac{1}{2}\, EB \,.\, AD = \frac{1}{2}\, AC \,.\, BD$.

$$\therefore sin\ (\alpha - \beta) = \frac{EB}{AB} = \frac{AC \,.\, BD}{AD \,.\, AB}$$

$$= \frac{AC(CD - CB)}{AD \,.\, AB} = \frac{CD \,.\, AC}{AD \,.\, AB} - \frac{AC \,.\, BC}{AD \,.\, AB}.$$

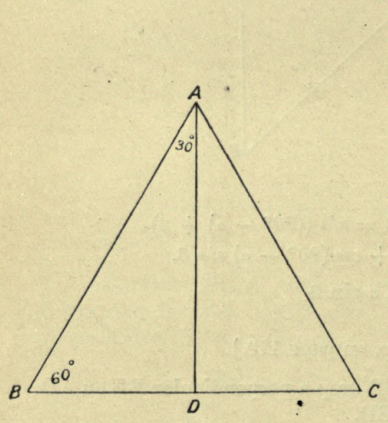
$$\therefore \mathbf{sin\ (\alpha - \beta) = sin\ \alpha\ cos\ \beta - sin\ \beta\ cos\ \alpha.}$$

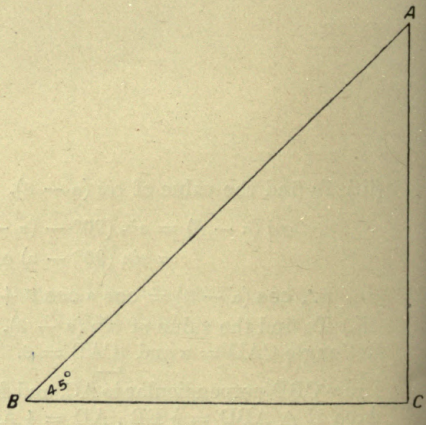



(iv.) To find the value of $cos\ (\alpha + \beta)$.

$$cos\ (\alpha + \beta) = sin\ (90° - (\alpha + \beta)) = sin\ ((90° - \alpha) - \beta)$$
$$= sin\ (90° - \alpha)\ cos\ \beta - cos\ (90° - \alpha)\ sin\ \beta.$$
$$\therefore \mathbf{cos\ (\alpha + \beta) = cos\ \alpha\ cos\ \beta - sin\ \alpha\ sin\ \beta.}$$

Corollary.—If $\alpha = \beta$ then $cos\ (\alpha + \beta) = cos\ 2\alpha = cos^2\ \alpha - sin^2\ \alpha = 2\ cos^2\ \alpha - 1$.




§ 7.—Evolution of some trigonometrical ratios.

(i.) In an equilateral △ each angle is 60°. Let ABC be such a △; drop a perpendicular AD from A on to BC.

$$\angle BAD = \tfrac{1}{2}\ 60° = 30°.$$

Let BD = 1, then AB = 2, and AD = $\sqrt{2^2 - 1^2} = \sqrt{3}$

$$\therefore \sin 60° = \cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$$

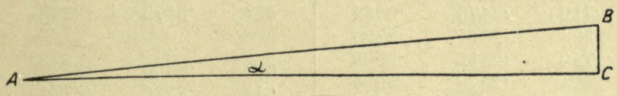
$$\cos 60° = \sin 30° = \frac{BD}{AB} = \frac{1}{2}$$

$$\tan 60° = \cot 30° = \frac{AD}{BD} = \sqrt{3}$$

$$\cot 60° = \tan 30° = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$$

(ii.) Let the sides AC and BC be equal in the △ ABC. Then ∠s CAB, ABC are each 45°.

$$\therefore \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}, \text{ and } \tan 45° = 1.$$




(iii.) If the angle $\alpha$ is very small

$$\tan \alpha = \frac{BC}{AC} = \frac{\text{a very small length}}{\text{a fairly big length}}$$

$$\sin \alpha = \frac{BC}{AB} = \frac{\text{a very small length}}{\text{a fairly big length}}$$

$$\cos \alpha = \frac{AC}{AB} = \text{the ratio of two nearly equal lengths.}$$

Hence, when $\alpha = 0$; $\tan \alpha = 0 = \sin \alpha$ and $\cos \alpha = 1$.

(iv.) $$\tan 90° = \cot 0° = \frac{1}{\tan 0°} = \frac{1}{0} = \infty$$

$$\sin 90° = \cos 0° = 1 \qquad \cos 90° = \sin 0° = 0.$$

By means of the formulæ proved in § 5 the ratios for 75° and 15° can be easily calculated.

§ 8.—$\sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta = 2\sin\alpha\cos\beta.$

If $\gamma = \alpha + \beta$ and $\delta = \alpha - \beta$, then $\gamma + \delta = 2\alpha$ and $\gamma - \delta = 2\beta$.

$$\therefore \sin\gamma + \sin\delta = 2\sin\frac{\gamma + \delta}{2}\cos\frac{\gamma - \delta}{2}.$$

## TABLE OF SINES OF ANGLES.

| | 0′ | 20′ | 40′ | | 0′ | 20′ | 40′ |
|---|---|---|---|---|---|---|---|
| 0° | ·0000 | ·0058 | ·0087 | 45° | ·7071 | ·7112 | ·7133 |
| 1° | ·0175 | ·0233 | ·0291 | 46° | ·7193 | ·7234 | ·7274 |
| 2° | ·0349 | ·0407 | ·0465 | 47° | ·7314 | ·7353 | ·7392 |
| 3° | ·0523 | ·0581 | ·0640 | 48° | ·7431 | ·7470 | ·7509 |
| 4° | ·0698 | ·0756 | ·0814 | 49° | ·7547 | ·7585 | ·7623 |
| 5° | ·0871 | ·0929 | ·0987 | | | | |
| 6° | ·1045 | ·1103 | ·1161 | 50° | ·7660 | ·7698 | ·7735 |
| 7° | ·1219 | ·1276 | ·1334 | 51° | ·7771 | ·7808 | ·7844 |
| 8° | ·1392 | ·1449 | ·1507 | 52° | ·7880 | ·7916 | ·7951 |
| 9° | ·1564 | ·1622 | ·1679 | 53° | ·7986 | ·8021 | ·8056 |
| | | | | 54° | ·8090 | ·8124 | ·8158 |
| 10° | ·1736 | ·1794 | ·1851 | 55° | ·8192 | ·8225 | ·8258 |
| 11° | ·1908 | ·1965 | ·2022 | 56° | ·8290 | ·8323 | ·8355 |
| 12° | ·2079 | ·2136 | ·2193 | 57° | ·8387 | ·8418 | ·8450 |
| 13° | ·2250 | ·2306 | ·2363 | 58° | ·8480 | ·8511 | ·8542 |
| 14° | ·2419 | ·2476 | ·2532 | 59° | ·8572 | ·8601 | ·8631 |
| 15° | ·2588 | ·2644 | ·2700 | | | | |
| 16° | ·2756 | ·2812 | ·2868 | 60° | ·8660 | ·8689 | ·8718 |
| 17° | ·2924 | ·2979 | ·3035 | 61° | ·8746 | ·8774 | ·8802 |
| 18° | ·3090 | ·3145 | ·3201 | 62° | ·8829 | ·8857 | ·8884 |
| 19° | ·3256 | ·3311 | ·3365 | 63° | ·8910 | ·8936 | ·8962 |
| | | | | 64° | ·8988 | ·9013 | ·9038 |
| 20° | ·3420 | ·3475 | ·3529 | 65° | ·9063 | ·9088 | ·9100 |
| 21° | ·3584 | ·3638 | ·3692 | 66° | ·9135 | ·9159 | ·9182 |
| 22° | ·3746 | ·3800 | ·3854 | 67° | ·9205 | ·9228 | ·9250 |
| 23° | ·3907 | ·3961 | ·4014 | 68° | ·9272 | ·9293 | ·9315 |
| 24° | ·4067 | ·4120 | ·4173 | 69° | ·9336 | ·9356 | ·9377 |
| 25° | ·4226 | ·4279 | ·4331 | | | | |
| 26° | ·4384 | ·4436 | ·4488 | 70° | ·9397 | ·9417 | ·9436 |
| 27° | ·4540 | ·4592 | ·4643 | 71° | ·9455 | ·9474 | ·9492 |
| 28° | ·4695 | ·4746 | ·4797 | 72° | ·9511 | ·9528 | ·9546 |
| 29° | ·4848 | ·4899 | ·4950 | 73° | ·9563 | ·9580 | ·9596 |
| | | | | 74° | ·9613 | ·9628 | ·9644 |
| 30° | ·5000 | ·5050 | ·5100 | 75° | ·9659 | ·9674 | ·9689 |
| 31° | ·5150 | ·5200 | ·5250 | 76° | ·9703 | ·9717 | ·9730 |
| 32° | ·5299 | ·5348 | ·5398 | 77° | ·9744 | ·9757 | ·9769 |
| 33° | ·5446 | ·5495 | ·5544 | 78° | ·9781 | ·9793 | ·9805 |
| 34° | ·5592 | ·5640 | ·5688 | 79° | ·9816 | ·9827 | ·9838 |
| 35° | ·5736 | ·5783 | ·5831 | | | | |
| 36° | ·5878 | ·5925 | ·5972 | 80° | ·9848 | ·9858 | ·9868 |
| 37° | ·6018 | ·6065 | ·6111 | 81° | ·9877 | ·9886 | ·9894 |
| 38° | ·6157 | ·6202 | ·6248 | 82° | ·9903 | ·9911 | ·9918 |
| 39° | ·6293 | ·6338 | ·6383 | 83° | ·9925 | ·9932 | ·9939 |
| | | | | 84° | ·9945 | ·9951 | ·9957 |
| 40° | ·6428 | ·6472 | ·6517 | 85° | ·9962 | ·9967 | ·9971 |
| 41° | ·6561 | ·6604 | ·6648 | 86° | ·9976 | ·9980 | ·9983 |
| 42° | ·6691 | ·6734 | ·6777 | 87° | ·9986 | ·9989 | ·9992 |
| 43° | ·6820 | ·6862 | ·6905 | 88° | ·9994 | ·9996 | ·9997 |
| 44° | ·6947 | ·6988 | ·7030 | 89° | ·9998 | ·9999 | 1·0000 |

## COSINES OF ANGLES.

| | 0′ | 20′ | 40′ |
|---|---|---|---|
| 0° | 1·0000 | 1·0000 | ·9999 |
| 1° | ·9998 | ·9997 | ·9996 |
| 2° | ·9994 | ·9992 | ·9989 |
| 3° | ·9986 | ·9983 | ·9980 |
| 4° | ·9976 | ·9971 | ·9967 |
| 5° | ·9962 | ·9957 | ·9951 |
| 6° | ·9945 | ·9939 | ·9932 |
| 7° | ·9925 | ·9918 | ·9911 |
| 8° | ·9903 | ·9894 | ·9886 |
| 9° | ·9877 | ·9868 | ·9858 |
| 10° | ·9848 | ·9838 | ·9827 |
| 11° | ·9816 | ·9805 | ·9793 |
| 12° | ·9781 | ·9769 | ·9757 |
| 13° | ·9744 | ·9730 | ·9717 |
| 14° | ·9703 | ·9689 | ·9674 |
| 15° | ·9659 | ·9644 | ·9628 |
| 16° | ·9613 | ·9596 | ·9580 |
| 17° | ·9563 | ·9546 | ·9528 |
| 18° | ·9511 | ·9492 | ·9474 |
| 19° | ·9455 | ·9436 | ·9417 |
| 20° | ·9397 | ·9377 | ·9356 |
| 21° | ·9336 | ·9315 | ·9293 |
| 22° | ·9272 | ·9250 | ·9228 |
| 23° | ·9205 | ·9182 | ·9159 |
| 24° | ·9135 | ·9100 | ·9088 |
| 25° | ·9063 | ·9038 | ·9013 |
| 26° | ·8988 | ·8962 | ·8936 |
| 27° | ·8910 | ·8884 | ·8857 |
| 28° | ·8829 | ·8802 | ·8774 |
| 29° | ·8746 | ·8718 | ·8689 |
| 30° | ·8660 | ·8631 | ·8601 |
| 31° | ·8572 | ·8542 | ·8511 |
| 32° | ·8480 | ·8450 | ·8418 |
| 33° | ·8387 | ·8355 | ·8323 |
| 34° | ·8290 | ·8258 | ·8225 |
| 35° | ·8192 | ·8158 | ·8124 |
| 36° | ·8090 | ·8056 | ·8021 |
| 37° | ·7986 | ·7951 | ·7916 |
| 38° | ·7880 | ·7844 | ·7808 |
| 39° | ·7771 | ·7735 | ·7698 |
| 40° | ·7660 | ·7623 | ·7585 |
| 41° | ·7547 | ·7509 | ·7470 |
| 42° | ·7431 | ·7392 | ·7353 |
| 43° | ·7314 | ·7274 | ·7234 |
| 44° | ·7193 | ·7133 | ·7112 |

| | 0′ | 20′ | 40′ |
|---|---|---|---|
| 45° | ·7071 | ·7030 | ·6988 |
| 46° | ·6947 | ·6905 | ·6862 |
| 47° | ·6820 | ·6777 | ·6734 |
| 48° | ·6691 | ·6648 | ·6604 |
| 49° | ·6561 | ·6517 | ·6472 |
| 50° | ·6428 | ·6383 | ·6338 |
| 51° | ·6293 | ·6248 | ·6202 |
| 52° | ·6157 | ·6111 | ·6065 |
| 53° | ·6018 | ·5972 | ·5925 |
| 54° | ·5878 | ·5831 | ·5783 |
| 55° | ·5736 | ·5688 | ·5640 |
| 56° | ·5592 | ·5544 | ·5495 |
| 57° | ·5446 | ·5398 | ·5348 |
| 58° | ·5299 | ·5250 | ·5200 |
| 59° | ·5150 | ·5100 | ·5050 |
| 60° | ·5000 | ·4950 | ·4899 |
| 61° | ·4848 | ·4797 | ·4746 |
| 62° | ·4695 | ·4643 | ·4592 |
| 63° | ·4540 | ·4488 | ·4436 |
| 64° | ·4384 | ·4331 | ·4279 |
| 65° | ·4226 | ·4173 | ·4120 |
| 66° | ·4067 | ·4014 | ·3961 |
| 67° | ·3907 | ·3854 | ·3800 |
| 68° | ·3746 | ·3692 | ·3638 |
| 69° | ·3584 | ·3529 | ·3475 |
| 70° | ·3420 | ·3365 | ·3311 |
| 71° | ·3256 | ·3201 | ·3145 |
| 72° | ·3090 | ·3035 | ·2979 |
| 73° | ·2924 | ·2868 | ·2812 |
| 74° | ·2756 | ·2700 | ·2644 |
| 75° | ·2588 | ·2532 | ·2476 |
| 76° | ·2419 | ·2363 | ·2306 |
| 77° | ·2250 | ·2193 | ·2136 |
| 78° | ·2079 | ·2022 | ·1965 |
| 79° | ·1908 | ·1851 | ·1794 |
| 80° | ·1736 | ·1679 | ·1622 |
| 81° | ·1564 | ·1507 | ·1449 |
| 82° | ·1392 | ·1334 | ·1276 |
| 83° | ·1219 | ·1161 | ·1103 |
| 84° | ·1045 | ·0987 | ·0929 |
| 85° | ·0871 | ·0814 | ·0756 |
| 86° | ·0698 | ·0640 | ·0581 |
| 87° | ·0523 | ·0465 | ·0407 |
| 88° | ·0349 | ·0291 | ·0233 |
| 89° | ·0175 | ·0087 | ·0058 |

## TANGENTS OF ANGLES.

| | 0′ | 20′ | 40′ | | 0′ | 20′ | 40′ |
|---|---|---|---|---|---|---|---|
| 0° | ·0000 | ·0058 | ·0116 | 45° | 1·0000 | 1·0117 | 1·0235 |
| 1° | ·0175 | ·0233 | ·0291 | 46° | 1·0355 | 1·0477 | 1·0599 |
| 2° | ·0349 | ·0407 | ·0466 | 47° | 1·0724 | 1·0850 | 1·0977 |
| 3° | ·0524 | ·0582 | ·0641 | 48° | 1·1106 | 1·1237 | 1·1369 |
| 4° | ·0699 | ·0759 | ·0816 | 49° | 1·1504 | 1·1640 | 1·1778 |
| 5° | ·0875 | ·0934 | ·0992 | | | | |
| 6° | ·1051 | ·1110 | ·1169 | 50° | 1·1918 | 1·2059 | 1·2203 |
| 7° | ·1228 | ·1287 | ·1346 | 51° | 1·2349 | 1·2497 | 1·2647 |
| 8° | ·1405 | ·1465 | ·1524 | 52° | 1·2799 | 1·2954 | 1·3111 |
| 9° | ·1584 | ·1644 | ·1703 | 53° | 1·3270 | 1·3432 | 1·3597 |
| | | | | 54° | 1·3764 | 1·3934 | 1·4106 |
| 10° | ·1763 | ·1823 | ·1883 | 55° | 1·4281 | 1·4460 | 1·4641 |
| 11° | ·1944 | ·2004 | ·2065 | 56° | 1·4826 | 1·5013 | 1·5204 |
| 12° | ·2126 | ·2186 | ·2247 | 57° | 1·5399 | 1·5597 | 1·5798 |
| 13° | ·2309 | ·2370 | ·2432 | 58° | 1·6003 | 1·6212 | 1·6426 |
| 14° | ·2493 | ·2555 | ·2617 | 59° | 1·6643 | 1·6864 | 1·7090 |
| 15° | ·2679 | ·2742 | ·2805 | | | | |
| 16° | ·2867 | ·2931 | ·2994 | 60° | 1·7320 | 1·7556 | 1·7796 |
| 17° | ·3057 | ·3121 | ·3185 | 61° | 1·8040 | 1·8291 | 1·8546 |
| 18° | ·3249 | ·3314 | ·3378 | 62° | 1·8807 | 1·9074 | 1·9347 |
| 19° | ·3443 | ·3508 | ·3574 | 63° | 1·9626 | 1·9912 | 2·0204 |
| | | | | 64° | 2·0503 | 2·0809 | 2·1123 |
| 20° | ·3640 | ·3706 | ·3772 | 65° | 2·1445 | 2·1775 | 2·2113 |
| 21° | ·3839 | ·3906 | ·3973 | 66° | 2·2460 | 2·2817 | 2·3183 |
| 22° | ·4040 | ·4108 | ·4176 | 67° | 2·3559 | 2·3945 | 2·4342 |
| 23° | ·4245 | ·4314 | ·4383 | 68° | 2·4751 | 2·5171 | 2·5605 |
| 24° | ·4452 | ·4522 | ·4592 | 69° | 2·6051 | 2·6511 | 2·6985 |
| 25° | ·4663 | ·4734 | ·4806 | | | | |
| 26° | ·4877 | ·4950 | ·5022 | 70° | 2·7475 | 2·7980 | 2·8502 |
| 27° | ·5095 | ·5169 | ·5243 | 71° | 2·9042 | 2·9600 | 3·0178 |
| 28° | ·5317 | ·5392 | ·5467 | 72° | 3·0777 | 3·1397 | 3·2041 |
| 29° | ·5543 | ·5619 | ·5696 | 73° | 3·2709 | 3·3402 | 3·4124 |
| | | | | 74° | 3·4874 | 3·5656 | 3·6470 |
| 30° | ·5773 | ·5851 | ·5930 | 75° | 3·7320 | 3·8208 | 3·9136 |
| 31° | ·6009 | ·6088 | ·6168 | 76° | 4·0108 | 4·1126 | 4·2193 |
| 32° | ·6249 | ·6330 | ·6412 | 77° | 4·3315 | 4·4494 | 4·5736 |
| 33° | ·6494 | ·6577 | ·6661 | 78° | 4·7046 | 4·8430 | 4·9894 |
| 34° | ·6745 | ·6830 | ·6916 | 79° | 5·1446 | 5·3093 | 5·4845 |
| 35° | ·7002 | ·7089 | ·7177 | | | | |
| 36° | ·7265 | ·7355 | ·7445 | 80° | 5·6713 | 5·8708 | 6·0844 |
| 37° | ·7536 | ·7627 | ·7720 | 81° | 6·3138 | 6·5606 | 6·8269 |
| 38° | ·7813 | ·7907 | ·8002 | 82° | 7·115 | 7·429 | 7·770 |
| 39° | ·8098 | ·8195 | ·8292 | 83° | 8·144 | 8·556 | 9·010 |
| | | | | 84° | 9·514 | 10·078 | 10·712 |
| 40° | ·8391 | ·8491 | ·8591 | 85° | 11·430 | 12·250 | 13·197 |
| 41° | ·8693 | ·8796 | ·9899 | 86° | 14·301 | 15·605 | 17·169 |
| 42° | ·9004 | ·9110 | ·9217 | 87° | 19·08 | 21·47 | 24·54 |
| 43° | ·9325 | ·9435 | ·9545 | 88° | 28·64 | 34·37 | 42·96 |
| 44° | ·9657 | ·9770 | ·9884 | 89° | 57·29 | 85·94 | 171·89 |

$\pi$ = 3·1416.
1 radian = 57°·296.
1° = 0·01745 radian.

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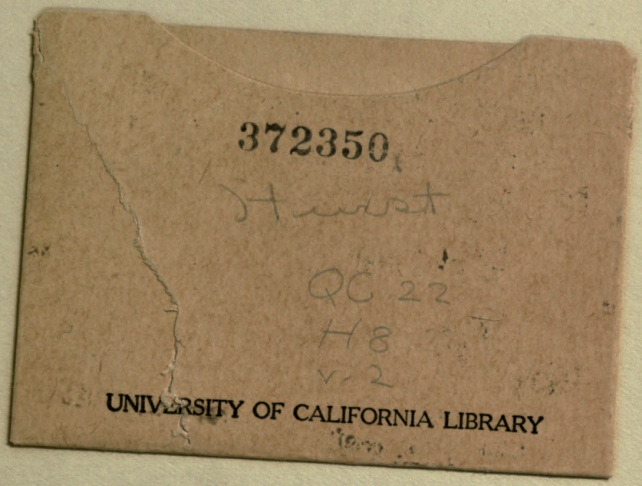
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